

The Value of Information is driven by the Intensity of Preference

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September 2006

Abstract

According to previous research, the value of information about a risky decision does not seem to depend in any consistent or predictable fashion on the features of the decision problem. For instance, the relationship between value of information and the degree of risk aversion can be non-monotonic, seemingly counterintuitive in some cases. This paper shows that value of information obeys a simple and general principle. The value of information is systematically driven by the intensity of preference between the decision alternatives: it will be maximal when the decision maker is indifferent between the prior alternatives, and lower as the preference for one of the alternatives is stronger. This result holds for any given choice alternatives, information structure, random background wealth, and utility function. The result bears some intuition. It also indicates that *quasi-concave* relationships should generally be expected between value of information and specific parameters of the decision problem.

Key words: value of information, expected utility, decision analysis, risk aversion

Suppose that your company is trying to decide which of two target firms, Company *X* or Company *Y*, to acquire. Both acquisitions involve substantial risk and the decision is far from obvious. As it stands now, your overall valuation of *X* comes out on top. There may be some value in further investigating the uncertainties surrounding the two acquisitions. Meanwhile you just learn that Company *Y* won a new contract, which brings its valuation closer to that of *X*. Do you now have more or less incentive to collect additional information before deciding? Because your decision has just become less clear-cut, you may feel that your value for further information is now higher. Now, if Company *Y* was initially your higher valued prospect, the news that it just got better would probably comfort you in your tentative choice and may decrease your need for further information. This paper shows that, under Expected Utility (EU) theory, definite answers can be provided to these questions in an extremely general sense. Yet, the theoretical result presented is a priori not evident given that value of information has been shown to vary in unexpected, sometimes plainly counterintuitive ways.

1. Lack of clear relationships between value of information and decision setting

The principles for valuing information are well established in EU theory. The value of information is linked to the possible increase in expected utility accrued by obtaining the information. More accurate information can be shown to have higher value than less accurate information (Blackwell, 1953). This readily implies two results: for any given decision situation, (1) costless information has non-negative value, and (2) *perfect* information has more value than any other information. Another standard result is that information can have positive value only if it is liable to change the default decision that would be made in the absence of the information.

Beyond these classic results, research has attempted to derive general principles on how value of information may depend on parameters of the problem setting. The puzzling finding is that no further general claim seems possible. For instance, previous authors have examined the relationship between value of information and risk aversion (e.g., Freixas & Kihlstrom, 1984). Since gathering information before acting is a means to avoid some of the risks inherent in a decision, intuition would seem to indicate that higher risk aversion will induce a higher valuation of information for any given situation. However, this is not always true: value of information may *decrease* as risk aversion increases. Eeckhoudt & Godfroid (2000) observe that excess use of medical diagnostic tests is often attributed to physicians' risk aversion, whereas, in theory, there is no evident link between risk aversion and demand for information, which they demonstrate by way of an example.

In a comprehensive review, Hilton (1981) endeavored to summarize results on the determinants of value of information. For this, he examined several features of the decision context, and concluded that ‘While changes in the decision maker’s flexibility, risk attitude, wealth and initial uncertainty have a significant effect on information value, their effect is not consistent across decision makers or decision settings.’ Specifically, Hilton (1981) referred to the fact that there are no systematic relationships between information value and key characteristics of the decision problem. Hazen & Sounderpandian (1999) briefly review some other examples of the unwieldiness of information value, noting that it is ‘notorious for its lack of convenient mathematical properties.’

Here we show that a fundamental determinant of value of information (VOI) is the intensity of preference that the decision maker entertains over the prior, uninformed decision alternatives. In a given decision situation, VOI will be highest if the decision maker is exactly indifferent between the prior choice alternatives, and it will be lower as the preference for one alternative over the other is stronger. This pattern holds under the most general conditions, and it provides a key to understanding the elusive nature of relationships between VOI and specific parameters of the decision problem. Section 2 presents the main result of the paper in terms of EU, and Section 3 examines how this result translates in terms of willingness to pay for information. Section 4 concludes with brief remarks.

2. Value of Information vs. Intensity of Preference

Consider an individual faced with the choice between two gambles, and the possibility of obtaining additional information at no cost. The problem elements and notation are as follows:

Decision maker: The individual decision maker (DM) is an EU maximizer with current wealth W and continuous utility function u . W may be stochastic, that is, the individual may own some unresolved background gambles.

Decision alternatives: The DM faces a choice between two alternatives, X and Y . The probability distributions of X and Y can be any, and they need not be specified. In particular, Y could be the status quo alternative, that is, the DM’s choice is simply between accepting or rejecting X .

Information: Information may be obtained by observing the value of a random variable Z , which may be correlated with X and Y . Observing the value of Z would provide imperfect information on *both* X and Y . Furthermore, Z could be correlated with W , in which case it would provide information about the uncertain background wealth.

Figure 1 represents the decision situation in the form of a decision tree. The DM may choose to add X or Y to his/her wealth W , depending on which provides greater utility. In the absence of information,

the DM's expected utility is $\max(Eu(W+X), Eu(W+Y))$ ¹. Obtaining information by observing the value of Z may further increase the DM's utility. Upon observing the value of Z , the DM will be faced with the choice between $(W+X|Z)$ and $(W+Y|Z)$, where $(W+X|Z)$ denotes gamble $W+X$ whose probability distribution has been updated in a Bayesian fashion given the observation of the value of Z , consistent with the standard notation of conditional probabilities. Given Z , the DM will choose X if $Eu(W+X|Z) \geq Eu(W+Y|Z)$, and Y otherwise, thus having utility $\max(Eu(W+X|Z), Eu(W+Y|Z))$. Taking the expectation of this over all possible values of Z , the utility of observing Z is thus: $E_Z[\max(Eu(W+X|Z), Eu(W+Y|Z))]$.

The value of information is measured as the increase in utility generated by information over the uninformed alternatives (Marschak & Radner, 1972; Hilton, 1981), that is:²

$$VOI = E_Z[\max(Eu(W+X|Z), Eu(W+Y|Z))] - \max(Eu(W+X), Eu(W+Y)). \quad (1)$$

Clearly, VOI depends, in possibly intricate ways, on any of the parameters entering Equation (1), including the probability distributions of X , Y and W ; the structure of correlation between Z , X , Y , and W ; the decision maker's utility function u . Beneath these complexities, we will show here that VOI is driven at a very basic level by the intensity of preference between the choice alternatives, X , Y , as measured by their difference in utilities $(Eu(W+X) - Eu(W+Y))$.

To show this, we have to consider X and Y being more or less distant in utility, keeping everything else constant. Note that the situation is symmetric with respect to X and Y . We will hold one of the two gambles, arbitrarily X , fixed, and replace Y by Y' such that the intensity of preference between X and Y' is strictly higher (or lower) than that between X and Y , *regardless* of the realization of Z . We will then examine how this affects VOI. Formally, Y' is such that $Eu(W+Y'|Z) > Eu(W+Y|Z)$ for all values of Z . This implies $Eu(W+Y') > Eu(W+Y)$. Also, this will not materially alter the original information structure, in the following sense: if $(W+Y|Z)$ is preferred to $(W+X|Z)$ for some value of Z , then replacing Y by a better Y' (i.e., with higher utility) should *not* reverse this preference. A straightforward way to come up with a qualified Y' , if u is monotonic, would be to add a constant to Y , i.e., translate it. This would furthermore preserve the correlation structure among all random variables involved. However, we need not go that far; we only require the weaker condition that, relative to Y , Y' should be closer to, or further from, X in utility, for all possible information messages.

¹ The + sign inside $u(\cdot)$ brackets refers to the operation of compounding gambles. $Eu(W+X)$ denotes the expected utility of the compound gamble $W+X$; we drop the brackets of the expectation operator to avoid a proliferation of brackets in the larger mathematical formulas.

² Hazen & Souderpandian (1999) consider other measures, including the increase in Certainty Equivalent and the Selling Price of information, and show that these measures rank information sources consistently, except for the Buying Price for information. Information Buying Price (or willingness to pay) will be addressed in Sect. 3.

The central result of the paper may now be stated as follows.

Theorem 1. For any given gambles X , Y , wealth W , utility function u , and information source Z , if the value of information VOI as defined in (1) has non zero value, then it is

- (a) strictly decreasing with $Eu(W+X) - Eu(W+Y) \geq 0$, i.e., if X is preferred to Y ;
- (b) strictly decreasing with $Eu(W+Y) - Eu(W+X) \geq 0$, i.e., if Y is preferred to X ;
- (c) maximal for $Eu(W+X) - Eu(W+Y) = 0$.

Proof. Let us consider the cases $Y \preceq X$ and $Y \succ X$ separately.

▪ **Case of X preferred to Y : $Eu(W+Y) < Eu(W+X)$**

In this case, (1) becomes: $VOI = E_Z [\max(Eu(W+X|Z), Eu(W+Y|Z))] - Eu(W+X)$. By observing that $Eu(W+X) = E_Z [Eu(W+X|Z)]$, this can be rewritten as:

$$\begin{aligned} VOI &= E_Z [\max(Eu(W+X|Z), Eu(W+Y|Z))] - E_Z [Eu(W+X|Z)] \\ &= E_Z [\max(Eu(W+X|Z), Eu(W+Y|Z)) - Eu(W+X|Z)] \\ &= E_Z [\max(0, Eu(W+Y|Z) - Eu(W+X|Z))]. \end{aligned} \quad (2)$$

Consider Y' such that $Eu(W+Y) < Eu(W+Y') \leq Eu(W+X)$ and $Eu(W+Y|Z) < Eu(W+Y'|Z)$ for any value of Z . The intensity of preference for X over Y' is strictly less than that for X over Y , for any value of Z . Since $VOI > 0$, there exists some value z of Z such that $Eu(W+Y|Z=z) - Eu(W+X|Z=z) > 0$, that is, although X is preferred to Y prior to information, Y will be chosen over X for some information signal z . For such values of Z , we have:

$$\max(0, Eu(W+Y|Z) - Eu(W+X|Z)) < \max(0, Eu(W+Y'|Z) - Eu(W+X|Z));$$

for all other values of Z the same inequality holds but it is non strict. Therefore, we have:

$$E_Z [\max(0, Eu(W+Y|Z) - Eu(W+X|Z))] < E_Z [\max(0, Eu(W+Y'|Z) - Eu(W+X|Z))].$$

The right side of the preceding inequality is precisely the value of information, VOI' , when Y' is substituted for Y . Thus we have $VOI < VOI'$. This shows that, for any $Y \preceq X$, VOI strictly increases with $Eu(W+Y)$, that is, it strictly decreases with $(Eu(W+X) - Eu(W+Y))$.

▪ **Case of Y preferred to X : $Eu(W+X) \leq Eu(W+Y)$**

Equation (1) now becomes: $VOI = E_Z [\max(Eu(W+X|Z), Eu(W+Y|Z))] - Eu(W+Y)$. Because $Eu(W+Y) = E_Z [Eu(W+Y|Z)]$, similarly as before, VOI can be re-expressed as:

$$\text{VOI} = E_Z \left[\max(Eu(W+X|Z) - Eu(W+Y|Z), 0) \right]. \quad (3)$$

Consider Y' such that $Eu(W+X) \leq Eu(W+Y) < Eu(W+Y')$ and $Eu(W+Y|Z) < Eu(W+Y'|Z)$ for all values of Z . The intensity of preference for Y' over X is *strictly larger* than that for Y over X . With Y' , the value of information becomes: $\text{VOI}' = E_Z \left[\max(Eu(W+X|Z) - Eu(W+Y'|Z), 0) \right]$. Because $\text{VOI} > 0$, there exists some value z of Z such that $Eu(W+X|Z=z) - Eu(W+Y|Z=z) > 0$, that is, X will be chosen over Y for some information signal z . For such values of Z , it is clear that $\max(Eu(W+X|Z) - Eu(W+Y'|Z), 0) < \max(Eu(W+X|Z) - Eu(W+Y|Z), 0)$; for all other values of Z , this inequality is non strict. Thus,

$$\begin{aligned} \text{VOI}' &= E_Z \left[\max(Eu(W+X|Z) - Eu(W+Y'|Z), 0) \right] \\ &< E_Z \left[\max(Eu(W+X|Z) - Eu(W+Y|Z), 0) \right] = \text{VOI}. \end{aligned}$$

In other words, an increase in the intensity of preference caused VOI to decrease.

In sum, the above shows that, for $Eu(W+X)$ fixed and different levels of $Eu(W+Y)$, whenever VOI is non zero VOI strictly increases with $Eu(W+Y)$ for $Y < X$, that is, *decreases* with the intensity of preference $Eu(W+X) - Eu(W+Y) \geq 0$; and it strictly decreases with $Eu(W+Y)$ for $Y \geq X$, that is, again, *decreases* with the intensity of preference $Eu(W+Y) - Eu(W+X) \geq 0$. Therefore, VOI must be maximum for $Eu(W+Y) = Eu(W+X)$. ■

Figure 2 gives a graphical account of the pattern of VOI as demonstrated above. It shows the pattern of the utilities of the three decision alternatives: 1) Take X , $Eu(W+X)$; 2) Take Y , $Eu(W+Y)$; 3) Acquire information before choosing, $E_Z \left[\max(Eu(W+X|Z), Eu(W+Y|Z)) \right]$; all as a function of $Eu(W+Y)$. The value of information is the vertical distance between $E_Z \left[\max(Eu(W+X|Z), Eu(W+Y|Z)) \right]$ and $\max(Eu(W+X), Eu(W+Y))$. If there exists any domain over which $\text{VOI} \neq 0$, then VOI will be strictly increasing for $Y \preccurlyeq X$, strictly decreasing for $Y \succcurlyeq X$, and reach a maximum at the break point of the $\max(\cdot)$ switch in Equation (1), that is, where the DM is indifferent between X and Y . The graph was generated using the following parameters. X and Y are binary gambles on a single event A : $X = (+0.8, A; -0.4, \text{not}A)$; $Y = (-0.3, A; +0.4, \text{not}A)$; $P[A] = 0.5$; Z provides imperfect information on A with two outcomes: 'predict A ', 'predict not A ', with $P[\text{predict } A|A] = 0.7$, $P[\text{predict } A|\text{not}A] = 0.2$. $W = 1$; utility of total wealth is: $u(x) = \text{Ln}(x)$. Finally Y was translated by adding a constant ranging from -0.4 to $+0.7$ to its payoffs. Let us underscore that the pattern of the graph is generic, that is, any other choice of parameters would produce a similar graph.

The intuition underlying the result may be articulated as follows. Assume the DM prefers X to Y before the information, should any information signal lead him/her to switch to Y , then the better Y is,

i.e. the closer in preference to X , the more the DM would benefit from this switch, and hence from the information. If, on the other hand, the DM prefers Y to X , then improving Y would make the switch to X less valuable *relative* to the improved Y (the default alternative), whenever information leads to that switch.

For clarity, we chose to hold $Eu(W+X)$ and consider different levels for $Eu(W+Y)$. Yet, the result of Theorem 1 does *not* depend on holding one of the two alternatives fixed. It can be seen from Equation (2) that the first part of the proof would obtain by replacing X with X' worse than X (for any value of Z). That is, by *reducing* the utility of the better alternative (X) closer to that of the lower one (Y), VOI would *go up*. Likewise, in Equation (3), the same result on VOI would obtain if the intensity of preference was augmented by decreasing $Eu(W+X|Z)$ instead of, or in addition to, increasing $Eu(W+Y|Z)$. Therefore, Theorem 1 would come through in the same way if the gap in utility between X and Y was modified by changing the utility levels of *both* X and Y together, that is, by considering X', Y' such that $Y < Y' \preceq X' < X$, or $X > X' \succcurlyeq Y' > Y$ (depending on which of the prior alternatives is preferred).

Finally, note that the result of Theorem 1 imposes no restriction on the shape or properties of u itself, because it relies solely on changing the levels of utilities, *not* on how u varies with changes in its arguments. The utility function could be concave, convex, S-shaped, or even non-monotonic (e.g., inverted U-shaped, as for salt in food or dose of a drug).

3. Willingness to Pay for Information vs. Intensity of Preference

An operational measure of information value commonly used in decision analysis is the maximum willingness to pay for the information (WTPI), in units of the decision payoffs. It is a cost c such that the DM would be indifferent between acquiring the information at cost c and the best alternative in the absence of information. Formally, $WTPI = c$ such that:

$$E_Z [\max(Eu(W-c+X|Z), Eu(W-c+Y|Z))] = \max(Eu(W+X), Eu(W+Y)). \quad (4)$$

The above equation can be shown to have a unique solution in c for u strictly monotonic. In that case, we may alternatively restate the definition of WTPI in terms of certainty equivalents by applying $u^{-1}(\cdot)$ to Equation (4):

$$c: u^{-1}(E_Z [\max(Eu(W-c+X|Z), Eu(W-c+Y|Z))]) = \max(CE[W+X], CE[W+Y]), \quad (5)$$

where $CE[W+X] = u^{-1}(Eu(W+X))$ is the certainty equivalent of $W+X$, similarly for $CE[W+Y]$.

Is WTPI maximal when VOI is maximal, that is, when $Eu(W+Y) = Eu(W+X)$? The answer is generally no, unless the utility function displays constant risk attitude, that is, u is linear or exponential. A counterexample is provided in the Appendix with the logarithm utility function.

The case of linear utility, $u(x) = a \cdot x + b$ with $a \neq 0$, is straightforward. With u linear, Equation (4) readily yields $c = \text{VOI}$, with VOI defined by Equation (1) in which is $u(x) = x$. This is nothing but the difference in the expected values with and without information, commonly called Expected Value of Imperfect Information (EVII). Therefore, from Theorem 1, EVII will be maximal if the choice alternatives have equal expected values, and lower as the difference in expected values is larger.

Beyond the linear case, a result similar to Theorem 1 can be obtained on WTPI for a widely used form of utility: the exponential utility function. It is stated in Theorem 2.

Theorem 2. If the utility function u displays constant risk attitude (i.e., u is exponential), for any given gambles X , Y , background wealth W , and information source Z , whenever the value of information is non zero, then willingness to pay for information is:

- (a) strictly decreasing with $Eu(W+X) - Eu(W+Y) \geq 0$, i.e., when X is preferred to Y ;
 - (b) strictly decreasing with $Eu(W+Y) - Eu(W+X) \geq 0$, i.e., when Y is preferred to X ;
 - (c) maximal for $Eu(W+X) - Eu(W+Y) = 0$;
- for any translation in gamble Y .

Theorem 2 specifies that variations in $Eu(W+X) - Eu(W+Y)$ be obtained by translations in Y , rather than just increases or decreases in utility of Y . This is a stronger requirement than in Theorem 1.

Proof. The exponential utility, defined as: $u(x) = -\text{sign}(r)e^{-rx}$, where r is the coefficient characterizing the individual's level of risk aversion (or risk seeking, if $r < 0$), possesses the property of constant absolute risk aversion (CARA). The nature of this property is that the addition or subtraction of a constant to a gamble will augment or reduce the gamble's certainty equivalent by the same constant. Observing that c is subtracted from every possible outcome on the left side of Equation (5), we can therefore rewrite this equation as:

$$c: u^{-1}\left(\mathbb{E}_Z\left[\max\left(Eu(W+X|Z), Eu(W+Y|Z)\right)\right]\right) - c = \max\left(CE[W+X], CE[W+Y]\right),$$

$$\text{that is: } c = u^{-1}\left(\mathbb{E}_Z\left[\max\left(Eu(W+X|Z), Eu(W+Y|Z)\right)\right]\right) - \max\left(CE[W+X], CE[W+Y]\right). \quad (6)$$

In other words, for exponential utility, the willingness to pay for information is just the difference in certainty equivalents between the informed and the uninformed decisions. To examine how WTPI varies with the intensity of preference, we will assume u increasing. The case of u decreasing can be

worked similarly, keeping in mind that c would be negative (that is, it takes an increase in the payoffs to lower utility). Let us again consider two cases.

▪ **Case of X preferred to Y : $Eu(W+Y) < Eu(W+X)$**

From (6), WTPI is equal to: $c = u^{-1}(E_Z[\max(Eu(W+X|Z), Eu(W+Y|Z))]) - CE[W+X]$. Take Y' such that $Eu(W+Y) < Eu(W+Y') \leq Eu(W+X)$ and $Eu(W+Y|Z) < Eu(W+Y'|Z)$ for all values of Z . The intensity of preference for X over Y' is lower than that for X over Y . WTPI associated with Y' is $c' = u^{-1}(E_Z[\max(Eu(W+X|Z), Eu(W+Y'|Z))]) - CE[W+X]$. From the first part of the proof of Theorem 1 we infer that:

$$E_Z[\max(Eu(W+X|Z), Eu(W+Y'|Z))] > E_Z[\max(Eu(W+X|Z), Eu(W+Y|Z))].$$

Applying u^{-1} , strictly increasing, on both sides and subtracting $CE[W+X]$, it follows that $c' > c$.

▪ **Case of Y preferred to X : $Eu(W+X) \leq Eu(W+Y)$**

From (6), $WTPI = c = u^{-1}(E_Z[\max(Eu(W+X|Z), Eu(W+Y|Z))]) - CE[W+Y]$. Consider $Y' = Y + \delta$, $\delta > 0$, such that $Eu(W+X) \leq Eu(W+Y) < Eu(W+Y')$. WTPI associated with Y' is:

$$c' = u^{-1}(E_Z[\max(Eu(W+X|Z), Eu(W+Y+\delta|Z))]) - CE[W+Y+\delta]. \quad (7)$$

By using the CARA property twice successively in (7), we get:

$$\begin{aligned} c' &= u^{-1}(E_Z[\max(Eu(W+X|Z), Eu(W+Y+\delta|Z))]) - CE[W+Y] - \delta \\ &= u^{-1}(E_Z[\max(Eu(W+X-\delta|Z), Eu(W+Y|Z))]) - CE[W+Y]. \end{aligned} \quad (8)$$

Since $VOI > 0$, there exists some value(s) of Z such that $Eu(W+X|Z) > Eu(W+Y|Z)$. Since $Eu(W+X|Z) > Eu(W+X-\delta|Z)$, for these values of Z we have: $\max(Eu(W+X|Z), Eu(W+Y|Z)) = Eu(W+X|Z) > \max(Eu(W+X-\delta|Z), Eu(W+Y|Z))$. For all other values of Z , we have the non strict inequality: $\max(Eu(W+X|Z), Eu(W+Y|Z)) \geq \max(Eu(W+X-\delta|Z), Eu(W+Y|Z))$. Hence:

$$E_Z[\max(Eu(W+X|Z), Eu(W+Y|Z))] > E_Z[\max(Eu(W+X-\delta|Z), Eu(W+Y|Z))]. \quad (9)$$

Since u^{-1} is strictly increasing, (8) and (9) imply $c > c'$. This shows that, for translations in Y , WTPI gets strictly smaller if $Eu(W+Y) - Eu(W+X)$ increases.

In sum, WTPI increases with $Eu(W+Y) < Eu(W+X)$, and decreases with $Eu(W+Y) \geq Eu(W+X)$. Therefore, WTPI must be maximum for $Eu(W+Y) = Eu(W+X)$. ■

Decision theorists have long recognized the intractabilities introduced by information costs, and they have generally avoided these difficulties by assuming linear or separable utility. As La Valle (1968)

observed, authors in information value theory have, at a certain point, ‘...dropped the general analysis and specialized to linear utility’ (p. 266). De Groot (1970) separated the cost of observation from the decision utility and recognized that by doing so ‘...we are making an assumption, in effect, about the additivity of the statistician’s utilities. Essentially, all work in statistical decision theory is based on this additive form for the total risk, and we shall use this form throughout the book.’ (p. 142). Marschak & Radner (1972) discuss the mathematical difficulties introduced by subtracting the cost of information from ‘gross payoff’ (pp. 83-85), and they state:

‘... the utility assigned to this combination is not, in general, representable as a *difference* between some “utility of things achieved” and “utility of things sacrificed.” However, because of its simplicity, the assumption that such separation of achievements from sacrifices is possible has great methodological advantages, at least as an approximation.’

Thus, statisticians —and some practitioners alike, e.g., Ades et al. (2004)— have essentially resorted to approximating Equation (4) as follows (assuming u strictly increasing):

$$E_Z [\max(Eu(W+X|Z), Eu(W+Y|Z))] - u(c) = \max(Eu(W+X), Eu(W+Y)).$$

Under this approximation we would get $WTPI = c = u^{-1}(VOI)$. Since u^{-1} is also strictly increasing, the result of Theorem 1 for VOI would also hold for WTPI, that is: WTPI peaks at indifference and vanishes as the preference for one alternative is stronger.

Issues associated with information costs are manifold. The cost of information could be random as opposed to constant, because it may depend on the information message, for example. This greatly complicates the assessment of information value (see La Valle, 1968). Also, the cost of information may be in different units than the decision outcomes. An example is in medical testing (Eeckhoudt et al., 1984), where costs of testing may occur in the form of monetary costs but also in health risks associated with the test itself, which, moreover, could be delayed into the future. In such cases, a treatment of WTPI would require a thorough multiattribute utility analysis. Sometimes, information is obtained by just waiting for the resolution of an uncertainty. There, the cost of information is incurred in terms of missed decision opportunities, which may not be expressible as in Equation (4).

The exponential utility function is widely accepted as a good approximation of risk taking behavior in decision analysis (e.g., Howard, 1988, p. 689): it can offer a good approximation to other smooth utility forms, especially if risks are not too large relative to total wealth. Indeed, it is often necessary to capture nothing more than the degree of concavity of the utility function over a given range of payoffs: exponential utility may serve this purpose well while affording mathematical tractability. In perhaps the most comprehensive simulation study, Kirkwood (2004) concluded that using exponential utility as an approximation to other utility forms will generally lead to correct decisions and accurate

valuations in a wide variety of circumstances. Kirkwood (2004) also points to boundaries beyond which the approximation becomes questionable. In sum, if exponential utility is an acceptable approximation, then the result of Theorem 2 holds as an approximation for other utility functions.

4. Concluding Remarks

Theorem 1 establishes a simple and general principle governing the value of information. It holds for *any* utility function, whatever the initial (random) wealth of the decision maker, whatever the distributions of the alternatives, and whatever the information source and its correlation with the choice alternatives and background risks. This contrasts with the previously noted “unruliness” of value of information. This principle also holds for the willingness to pay for information in the case of linear or exponential utility, forms widely used in decision analysis. The result has some immediate implications, which we briefly mention below.

- *Understanding relationships between VOI and specific decision features.* Consider a monotonic change in an aspect of one alternative, say, some outcome or probability in Y . This will change $Eu(W+Y)$ monotonically, but, as Theorem 1 shows, this will not translate into a monotonic variation in information value, but rather a quasi-concave variation. The same holds for a change in risk aversion level: for given X and Y , monotonic variations in risk aversion level will translate into monotonic changes in $(Eu(W+X) - Eu(W+Y))$, but the latter will result in a non monotonic change in information value. VOI would reach a maximum for that risk aversion level that makes the DM indifferent between X and Y . Theorem 1 thus conveys an intuition for why value of information may *decrease* with risk aversion: the higher the level of risk aversion, the stronger the preference for the safer alternative, but, past a certain point, the lower the value of information. More generally, Theorem 1 provides a key to anticipating how information value is liable to vary with decision setting parameters: one should expect quasi-concave (or “caret” shaped) relationships between information value and any parameter that determines the distance between the alternatives’ utilities.

- *Information about (random) wealth and the value of side bets.* The source of information Z considered in the above analysis could bear entirely on uncertain wealth W . Thus, Theorem 1 indicates that information about *unresolved background* gambles will have maximal value whenever the DM is just indifferent toward the new choice opportunities he/she faces. Bell (1988) shows that it may be beneficial for a decision maker to spontaneously engage in an *independent* side bet before choosing between two alternatives, even if the side bet is not attractive in its own right. This seems at first counterintuitive, yet a side bet can be viewed as a clever way to capitalize on the value of

information. The DM artificially creates uncertainty in his/her wealth by taking a side bet (a priori undesirable for a risk averter), but then arranges to resolve this uncertainty, i.e., get perfect information before deciding, something always (weakly) desirable. Should the choice between the prior alternatives depend on the wealth level, then the value of resolving the wealth uncertainty may outweigh the negative impact of this uncertainty in the first place, leaving our DM strictly better off. Now, the result of Theorem 1 readily suggests refinements to Bell's (1988) result: (1) the value of a side bet will be *maximal* if it is designed to render the individual indifferent between the alternatives augmented by the side bet; (2) the side bet does *not* have to be independent of background wealth.

- *Extension to real option value.* A real option adds value to a decision opportunity in exactly the same way as perfect information does. The present result brings to light the fact that the value of a real option on a given risk will be maximum if the decision maker is indifferent between accepting the risk or not in the absence of the real option.

The value of information may depend widely on any aspect of the decision situation, in ways that sometimes seem unpredictable or not easy to grasp intuitively. The central result derived here offers a readily intuitive principle to gauge the benefits of seeking further information: assessing the intensity of preference may provide enough guidance. Indeed, you will have most to gain from information if you are most ambivalent about your choice; least to gain if your mind is already strongly set on one alternative, *whatever* the reason for this preference. This principle appears to find some support in the experimental evidence reported by Tykocinski & Ruffle (2003). These authors found that those subjects that were less confident about their choice between two alternatives were also more likely to wait for further information. Our result appears to further extend the notion that information value is tightly linked to the potential for the information to alter decisions: the stronger the prior preference for one of the alternatives, the less the potential for new information to reverse this preference, and the less its incremental value.

Acknowledgments. Thanks to Ioana Popescu for suggesting simplifications in the mathematical treatment of Section 1, and to Bob Winkler, Louis Eeckhoudt, Marc Le Menestrel, Peter Wakker, Jim Dyer, David Bell, and Steven Chick for sharing comments on previous versions of this work.

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Appendix

Consider a decision maker with wealth $W = 1$ and log utility function: $u(x) = \ln(W + x)$, where x is an incremental payoff. The DM faces a choice between a two outcome gamble $X = \{2, 0.5; -2/3, 0.5\}$ and $Y =$ a constant. The certainty equivalent of X is easily calculated to be: $CE[X] = 0$.

Assume that perfect information is available about the outcome of X . The gain in expected utility provided by this information is maximum when $Y = 0 = CE[X]$, as demonstrated in Section 1. In that case, the decision maker's willingness to pay for perfect information on X is: $c = 0.586$, in monetary units.

Now, take $Y = 0.200 > CE[X]$. It is easy to verify numerically that the willingness to pay for perfect information is now equal to: $c^* = 0.600 > 0.586$. In fact, one can also verify that c^* is the peak in value of information as Y varies throughout the relevant range of payoffs.

This example shows that the peak in willingness to pay for information about X does not occur for $Y \sim X$, but rather for $Y > X$.

Note that for someone with total wealth 1 (e.g. million \$), a 50-50 gamble for +2 or $-2/3$ is a rather dizzying proposition.

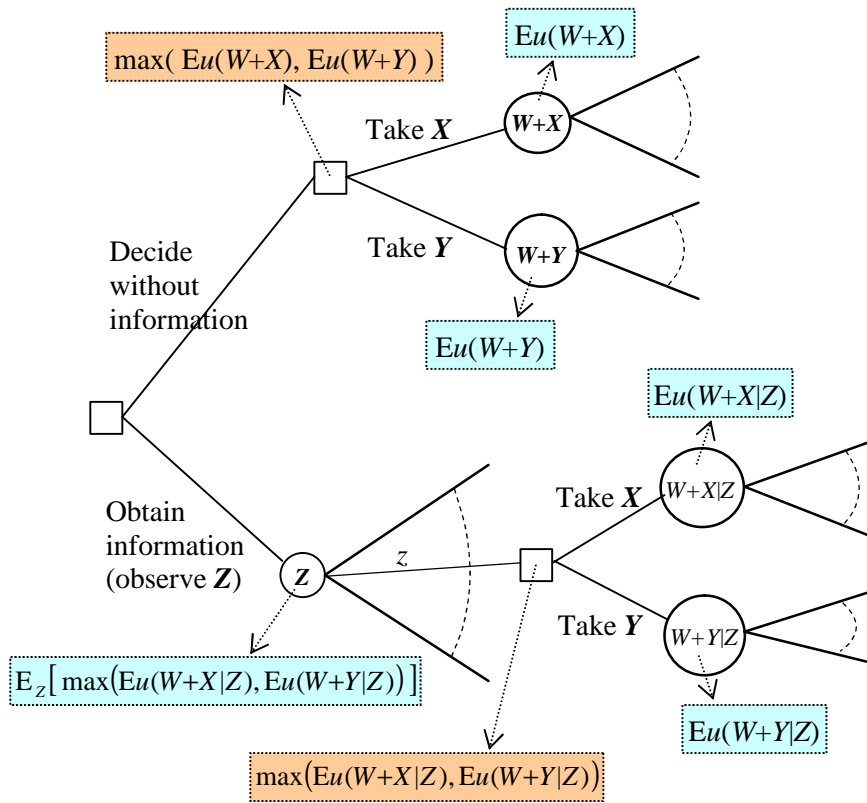


Figure 1. Decision tree representation of the value of information problem. Calculations at each node are shown in text boxes.

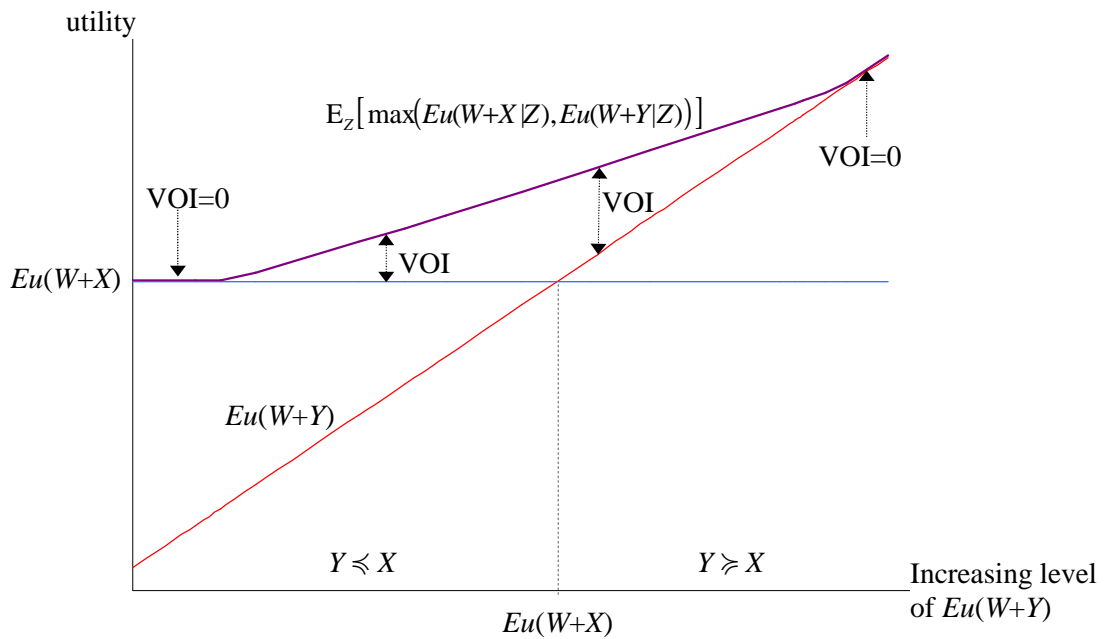


Figure 2. As the utility of one alternative takes on increasing levels, VOI goes through a maximum when both alternatives have equal utility.