Environmental externalities and efficiency measurement

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ABSTRACT.- *Production of desirable outputs is often accompanied by negative externalities on the environment. From the pioneering paper by Färe et al. (1989), environmental externalities have been mostly incorporated into traditional production theory and efficiency analysis considering that they are bad outputs under either weak or strong disposability. In this paper, we argue that, beyond the usefulness of treating environmental externalities as undesirable outputs, considering them as strongly disposable detrimental inputs could also be a useful empirical tool to test for the existence of technologies where the biggest producer is not the greatest polluter. Furthermore, we show that in such situations, environmental regulation could achieve an effective reduction in the aggregate level of bad outputs without reducing the production of good outputs. Finally, we illustrate our methodology with an empirical application to a sample of Spanish tile ceramic producers.*

Key words: *environmental externalities; efficiency measurement; weak disposability; detrimental inputs.*
JEL Classification: *C61; D21; L68.*

1. Introduction

Efficiency and productivity measurement is a longstanding matter of interest in economics. Furthermore, over the past two decades environmental performance and the assessment of the costs of abating negative environmental externalities that frequently go with production of desirable goods, are receiving growing attention by both academics and policy decision-makers. Environmental issues are also becoming a major matter of concern for firms' managers. According to this interest, from the eighties onward a growing literature has arisen devoted to incorporating environmental issues into traditional production theory, which has, to date, produced a wealth of contributions (see Tyteca, 1996 and Allen, 1999 for a review).

In this literature, it has been common practice to consider negative environmental externalities as undesirable or bad outputs into the production processes. Based on this assumption, two main approaches have been followed. On the one hand, some studies have focused on adjusting conventional productivity indexes to allow for *undesirables* into the output vector. Pitman (1983) pioneered this line of research by using econometric techniques to calculate shadow prices for undesirable outputs, which were used to compute an index of productivity change that accounts for environmental effects. Other papers in this line of research are Färe *et al.* (1993), Coggins and Swinton (1996), Swinton (1998) and Reig-Martínez *et al.* (2001). Furthermore, Chung *et al.* (1997) developed a *Malmquist-Luenberger* productivity index that credits producers for simultaneously increasing the production of desirable outputs while reducing the production of *undesirables*, also allowing the change in *Total Factor Productivity* to be decomposed into changes in efficiency and technical change. Subsequent papers in a similar line of research are Färe *et al.* (2001), Jeon and Sickles (2004), Domazlicky and Weber (2004) and Barla and Perelman (2005).

A second strand of the literature has addressed, on the other hand, the issue of modelling environmental externalities into the traditional efficiency analysis framework (Farrell, 1957). Färe *et al.* (1989) pioneered this line of research by considering that both good and bad outputs are weakly disposable, as a sensible way to model the idea that reducing *bads* has a cost of opportunity measurable as a lower production of good outputs. On this basis, a number of papers, including that by Chung *et al.* (1997) and Ball *et al.* (2001), have proposed the use of directional distance functions as an useful tool in modelling production theory in presence of *undesirables* and, particularly, in assessing environmental performance and calculating the costs of abating contamination. Furthermore, Färe *et al.* (2003) and Picazo-Tadeo *et al.* (2005) make use of directional technology distance functions to calculate productive-oriented efficiency measures, which are used to compute firm-specific reduced production of *goods* resulting from regulations that prevent free disposal of residuals. Other interesting papers in this line of research, are Ball *et al.* (1994), Brännlund *et al.* (1995), Färe *et al.* (1996), Tyteca (1997), Hernández-Sancho *et al.* (2000), Zofio and Prieto (2001) and Färe *et al.* (2005).

Over recent years, it has emerged a renewed interest in measuring environmental performance and pollution abatement costs arising from public regulation aimed at achieving determined standards of environmental quality. Opposite to the traditional approach of considering environmental externalities as bad outputs, some papers have adopted what we might call the *unconventional* view of considering environmental externalities as strongly disposable detrimental inputs in production processes. Pitman (1981) had already modelled *undesirables* as damaging inputs arguing that the relationship between any environmentally detrimental variable and desirable outputs is quite similar to the relationship that exists between conventional inputs and outputs. Other recent papers that adopt a similar view are Tyteca (1997), Reinhard *et al.* (1999, 2002), Hailu and Veeman (2001) and Prior (2005). Nonetheless, the assumption of considering environmental externalities as detrimental inputs has been criticised arguing that it might violate physical laws, because it could allow *unbounded* amounts of pollutants to be produced from limited quantities of inputs (Färe and Grosskopf, 2003).

In this manuscript we argue that, whereas considering environmental externalities as undesirable outputs under either weak or strong disposal has extensively demonstrated its usefulness in modelling environmental externalities into traditional production theory and efficiency measurement framework, treating environmental externalities as strongly disposable detrimental inputs might also reveal itself as a useful empirical tool to deal with particular production technologies. Mainly, we show that this axiom serves as an *instrumental* tool that helps to identify technologies where the biggest producer is not the greatest polluter, i.e., technologies belonging to the upward segment of the technological frontier in Färe *et at.* (1989). In addition, our paper provides meaningful economic arguments that support the possibility that these observation exist, which go beyond the conventional view of considering that they are simply outliers of measurement errors. Furthermore, we use directional distance functions to illustrate our methodological proposal with an empirical application to a sample of cross-section data of Spanish tile producers.

The remainder of the paper is organised as follows. Section two deals with methodological issues. Section three describes the data and discuses the empirical application. Finally, section four concludes.

2. Methodology

Introducing some notation, let us consider that production technology is defined by the set of feasible input, $\mathbf{x}=\left(x_1,...,x_i,...,x_I\right)\in\mathbf{R}_{+}^{I}$, and output, $\mathbf{y}=\left(y_1,...,y_j,...,y_J\right)\in\mathbf{R}_{+}^{J}$, vectors:

$$S=\left\{\left(\mathbf{x},\mathbf{y}\right)\mid \mathbf{x}\text{ can produce }\mathbf{y}\right\} \tag{1}$$

Traditional production theory assumes that the properties of the production technology are the following (Shephard, 1970; see also Grosskopf, 1986):

a) Inactivity.

a.1) *Possibility of inaction:* $\left(\mathbf{0},\mathbf{0}\right)\in S$ (2)

It is technological possible not to produce.

a.2) *No free lunch:* $(\mathbf{0},\mathbf{y})\notin S,\ \mathbf{y}>\mathbf{0}$ (3)

Positive quantities of inputs are required to produce positive amounts of outputs.

b) Strong disposability of inputs (SDI).

$$\forall\mathbf{x}\in\mathbf{R}_{+}^{I},\ \ if\ \left(\mathbf{x},\mathbf{y}\right)\in S\ \Rightarrow\ \left(\mathbf{x}',\mathbf{y}\right)\in S,\ \mathbf{x}'\geq\mathbf{x} \tag{4}$$

It is feasible to produce the same amount of output using a bigger quantity of any input, i.e., the inputs excess can be disposed at no cost.

c) Strong disposability of outputs (SDO).

$$\forall\mathbf{y}\in\mathbf{R}_{+}^{J},\ \ if\ \left(\mathbf{x},\mathbf{y}\right)\in S\ \Rightarrow\ \left(\mathbf{x},\mathbf{y}'\right)\in S,\ \mathbf{y}'\leq\mathbf{y} \tag{5}$$

Any feasible output can be freely disposed.

d) Convexity.

$$\begin{aligned}&\forall\mathbf{x},\mathbf{x}'\in\mathbf{R}_{+}^{I},\ \forall\mathbf{y},\mathbf{y}'\in\mathbf{R}_{+}^{J},\ \ if\ \left(\mathbf{x},\mathbf{y}\right),\left(\mathbf{x}',\mathbf{y}'\right)\in S\\&\Rightarrow\ \alpha\left(\mathbf{x},\mathbf{y}\right)+\left(1-\alpha\right)\left(\mathbf{x}',\mathbf{y}'\right)\in S,\ \ \alpha\in\left[0,1\right]\end{aligned} \tag{6}$$

The linear combinations of feasible production plans are also feasible.

Furthermore, let us assume that production of *desirable* outputs comes jointly with a set of polluting wastes that, by the moment, are considered to be undesirable or bad outputs. Thus, the output vector can be partitioned into a sub-vector of desirable or *good* outputs, $\mathbf{y}^g = \left(y_1^g, ..., y_g^g, ..., y_G^g \right) \in \mathbf{R}_+^G$, and a sub-vector of undesirable or *bad* outputs, $\mathbf{y}^b = \left(y_1^b, ..., y_b^b, ..., y_B^b \right) \in \mathbf{R}_+^B$, so that $\mathbf{y} = \left(\mathbf{y}^g, \mathbf{y}^b \right) \in \mathbf{R}_+^J$.

In order to introduce joint production of good and bad outputs into the characterisation of the technology, two additional axioms are needed, null-joint production and weak disposability of outputs, both desirable and undesirable. Formally, these axioms are formulated as:

e) Null-jointness.

$$\forall \mathbf{y}^g \in \mathbf{R}_+^G,\ \forall \mathbf{y}^b \in \mathbf{R}_+^B,\ \mathbf{y}^b = 0 \Rightarrow \mathbf{y}^g = 0 \tag{7}$$

f) Weak disposability of outputs (WDO).

$$\forall \mathbf{y}^g \in \mathbf{R}_+^G,\ \forall \mathbf{y}^b \in \mathbf{R}_+^B,\ \left(\mathbf{x}, \mathbf{y}^g, \mathbf{y}^b \right) \in S \Rightarrow \left(\mathbf{x}, \alpha \mathbf{y}^g, \alpha \mathbf{y}^b \right) \in S,\quad \alpha \le 1 \tag{8}$$

On the one hand, *null-jointness* was first introduced by Shephard and Färe (1974) to model the idea that good and bad outputs are jointly produced. This assumption means that if no bad outputs are produced then none of the good outputs are produced. Likewise, if some good outputs are produced, some quantity of undesirable outputs must also be produced.

Weak disposability of outputs assures, on the other hand, that proportional reduction of good and bad outputs is feasible, but the isolated disposal of *bads* may not be. *WDO* was proposed by Färe *et al.* (1989) as a reasonable way of recognising that reducing *undesirables* may not be a free activity, as traditional production theory assumes. Conversely, when firms face environmental-friendly regulations, disposing undesirable outputs involves a cost that could be measured in terms of *opportunity* as a shrinkage of the maximum attainable production of *goods* from a given endowment of resources (see also Färe and Primont, 1995). In other words, inputs that otherwise could have a productive use, i.e. production of desirable outputs, have to be diverted to reduce or eliminate undesirable outputs in compliance with the environmental regulations.

Our next theoretical building block is the *directional distance function* (*DDF*). This function generalises both input and output Shephard's distance functions, providing a complete representation of the production technology (Färe *et al.*, 2000 summarise the theory and main applications of these functions; see also Färe and Grosskopf, 2004). In presence of undesirable outputs, the *DDF* is defined as:

$$\vec{D}\left(\mathbf{x},\mathbf{y}^g,\mathbf{y}^b;-g_x,g_{y^g},-g_{y^b}\right)=\text{Sup}\left[\beta\,\middle|\left(\mathbf{x}-\beta g_x,\ \mathbf{y}^g+\beta g_{y^g},\ \mathbf{y}^b-\beta g_{y^b}\right)\in S\right] \qquad (9)$$

$\mathbf{g}=\left(-g_x,g_{y^g},-g_{y^b}\right)$ being the *direction vector*.

Directional distance functions model joint production of desirable and undesirable outputs, allowing for increasing *goods* while simultaneously reducing *bads*, along a path previously determined through a particular direction vector. Accordingly, the *DDF* of expression (9) inflates the vector of good outputs in the g_{y^g} direction and contracts the vectors of inputs and bad outputs in the $-g_x$ and $-g_{y^b}$ directions, respectively, while staying within the set of technologically feasible productive plans *S*. Furthermore, it can be proved that[1]:

$$\vec{D}\left(\mathbf{x},\mathbf{y}^g,\mathbf{y}^b;-g_x,g_{y^g},-g_{y^b}\right)\geq 0 \Leftrightarrow \left(\mathbf{x},\mathbf{y}^g,\mathbf{y}^b\right)\in S \qquad (10)$$

The computed *DDF* for a particular decision-making unit measures its level of technical efficiency[2]. If the *DDF* equals to zero, the productive unit under evaluation is efficient in the *Farrell-Debreu* notion[3]. In other words, no other peer has been found that yields greater vector of good outputs with little consumption of inputs and smaller level of bad outputs.

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[1] Other relevant properties of directional distance functions are summarised in Chambers *et al.* (1998).

[2] Computed scores of technical efficiency must be considered here as measures of *productive-oriented* efficiency, rather than output- or input-oriented efficiencies, given that the directional distance function expands *goods* simultaneously reducing inputs.

[3] Here, we follow the well-known *Farrell-Debreu* notion of efficiency. There exist, however, another more exigent concept of efficiency, the *Pareto-Koopmans* efficiency (see Färe *et al.*, 1994 for details).

Now, let us assume that we observe a sample of $k = 1,\dots,K$ decision-making units. Under the set of assumptions made concerning the technology, and imposing variable returns to scale[4], for a productive unit *k*, the *DDF* of expression (9) can be computed solving the following linear programming problem (see Färe *et al.*, 2003):

$$
\begin{aligned}
&\vec{D}_k^{WDB}\left(\mathbf{x}_k,\mathbf{y}_k^g,\mathbf{y}_k^b;-g_x,g_{y^g},-g_{y^b}\right)=Max\,\beta \\
&\text{subject to:} \\
&\left(\mathbf{x}_k-\beta g_x\right)\geq \mathbf{z}\,\mathbf{J} && (i) \\
&\left(\mathbf{y}_k^g+\beta g_{y^g}\right)\leq \delta\mathbf{z}\,\mathbf{G} && (ii) \\
&\left(\mathbf{y}_k^b-\beta g_{y^b}\right)=\delta\mathbf{z}\,\mathbf{B} && (iii) \\
&\mathbf{e}\mathbf{z}=1 && (iv) \\
&\mathbf{z}\geq 0 && (v) \\
&0\leq\delta\leq 1 && (vi)
\end{aligned}
\qquad (11)
$$

where:

- $\vec{D}_k^{WDB}$ is the computed *DDF* when *bads* are assumed weakly disposable (*WDB*),
- $\mathbf{x}_k$, $\mathbf{y}_k^g$ and $\mathbf{y}_k^b$ are, respectively, the vectors of inputs, good outputs and bad outputs corresponding to the productive unit under evaluation, i.e., firm *k*,
- **z** is an activity vector denoting the intensity at which observed productive units are conducted in constructing the technological frontier,
- **J, G** and **B** are three matrices containing the observed vectors of inputs, good outputs and bad outputs, respectively,
- **e** is a vector of ones, and, finally,
- δ is a parameter restricted to be within zero and one.

In program (11), the set of constraints in (*i*) guarantee that inputs are strongly disposable. Furthermore, the inequality constraints on the good outputs side in (*ii*) imply that these *goods* are freely disposable. Together with the strict equality constraints on the bad outputs side in (*iii*) and the parameter δ on the right hand side of restrictions (*ii*) and (*iii*), both good outputs

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[4] Other properties concerning the nature of the scale properties of the technology could also be assumed (see Banker *et al.*, 1984).

and bad outputs are weakly disposable when variable returns to scale are imposed. Finally, the model also satisfies null-jointness provided that (see Färe and Grosskopf, 2004):

$$\sum_{k=1}^{K} y_k^g > 0 \qquad g = 1,...,G \tag{12}$$

$$\sum_{g=1}^{G} y_k^g > 0 \qquad k = 1,...,K \tag{13}$$

Condition (12) implies that every undesirable output is produced by at least one productive unit, while condition (13) means that each firm produces at least one undesirable output[5].

Färe *et al.* (2003) refer to program (11) as *regulated*, provided that bad outputs can not be freely disposed. Conversely, when firms face no environmental rules Färe *et al.* (1989) proposed to characterise the technology throughout the axiom of strong disposability of undesirable outputs (*SDB*). That is:

g) Strong disposability of bad outputs (SDB).

$$\forall \mathbf{y}^b \in \mathbf{R}_+^B,\ \left(\mathbf{x}, \mathbf{y}^g, \mathbf{y}^b\right) \in S \Rightarrow \left(\mathbf{x}, \mathbf{y}^g, \mathbf{y}^b{}'\right) \in S,\ \mathbf{y}^b{}' \leq \mathbf{y}^b \tag{14}$$

In words, this assumption states that bad outputs can be reduced at no cost. This is a sagacious manner to model the idea that when no environmental legislation applies firms can freely dispose of polluting wastes. Nonetheless, it should be also noted that free disposal of wastes disrupts the physical link between good outputs and bad outputs, making the null-jointness hypothesis no longer necessary. Although the relationship between *goods* and *bads* always exists in the *real word*, in the unregulated scenario the assumption of *SDB* constitutes an *instrumental* tool that makes sense if it is interpreted in terms of *costs*, as proposed by Färe *et al.* (1989). That is, strong disposability is interpreted as the plausible assumption that in absence of regulation the firm will not bear the *cost* of disposing in a socially acceptable way of its undesirable output, rather than a *real possibility* of breaking the physical relationship between good outputs and bad outputs (see Picazo-Tadeo *et al.* 2005).

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[5] To further illustrate the relationship between null-jointness and these conditions, let us assume, for decision-making unit *k*, that each bad output in (11)-(*iii*) is equal to zero. In that case, because of conditions (12) and (13) each intensity variable must be zero, implying that every desirable output in (11)-(*ii*) must also be zero.

Under the assumption of *SDB*, the *unregulated* version of program (11) is:

$$
\begin{aligned}
&\vec{D}_k^{SDB}\left(\mathbf{x}_k, \mathbf{y}_k^g, \mathbf{y}_k^b; -g_x, g_{y^g}, -g_{y^b}\right) = Max\,\beta \\
&\text{subject to:} \\
&\left(\mathbf{x}_k - \beta g_x\right) \geq \mathbf{z}\,\mathbf{J} && (i) \\
&\left(\mathbf{y}_k^g + \beta g_{y^g}\right) \leq \mathbf{z}\,\mathbf{G} && (ii) \qquad\qquad (15) \\
&\left(\mathbf{y}_k^b - \beta g_{y^b}\right) \leq \mathbf{z}\,\mathbf{B} && (iii) \\
&\mathbf{ez} = 1 && (iv) \\
&\mathbf{z} \geq 0 && (v)
\end{aligned}
$$

where *SDB* has been introduced transforming the equality (11)-(*iii*) into the inequality (15)-(*iii*) and removing the parameter δ.

Figure 1 provides a graphic intuition of the evaluation of programs (11) and (15) based on a simple diagram. In order to make things easier, let us assume that all productive units present the same inputs vector to produce one good output and one bad output. Also, let us consider that the direction vector takes only the good output expansion direction. Formally, this direction vector is:

$$\mathbf{g} = \left(-g_x, g_{y^g}, -g_{y^b}\right) = \left(0,1,0\right) \qquad (16)$$

[Figure 1 about here]

Being productive unit *A* inefficient[6], by expanding to the maximum level of good output with the same level of bad output, the *DDF* computed assuming that environmental rules prevent free disposal of *bads*, i.e., the solution to program (11), locates point *A* on the boundary of the regulated technology at point $A' = A + \vec{D}_A^{WDB}$. In contrast, when there is not environmental regulation and firms can dispose of *undesirables* at no cost, program (15) finds point $A'' = A + \vec{D}_A^{SDB}$ as the benchmark for the good output of productive unit *A*. Nevertheless, as in the *real word* it is technologically infeasible for productive unit *A* to reach point *A"* (as noted, this would disrupt the physical relationship between good outputs and bad outputs), in absence of regulation the *real* bench-

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[6] Notice that, under the particular direction vector considered, technical inefficiencies should be interpreted as output-oriented inefficiencies, because this vector expands *goods* for given endowment of inputs.

mark for firm *A* is point *B* (note that $y_B^g = y_{\mathrm{A''}}^g$), but the strong disposability axiom, i.e., lack of environmental rules, permits firm *A* to freely dispose the increase in bad outputs, which is measured by $\left(y_B^b - y_A^b \right)$.

Having these results and following Picazo-Tadeo *et al.* (2005), we define the *regulatory impact index* as the difference between efficient projections of good output on both *regulated* and *unregulated* technological frontiers. Formally, for decision making unit *k*, this index is:

$$Regulatory\ impact\ index_k = \left(\vec{D}_k^{SDB} - \vec{D}_k^{WDB} \right) \qquad (17)$$

The regulatory impact index of expression (17) measures losses of good output due to regulation and takes values equal or greater than zero. Value zero means that environmental regulations are not binding and regulation is not hindering strong disposability of undesirable outputs. Conversely, a positive index indicates that regulation hinders free disposal of *bads*.

Let us assume now that the technology is that symbolized in *Figure 2,* where the biggest producer is not the biggest polluter, i.e., the output set is that depicted in Färe *et al.* (1989). In this new scenario, productive unit *B* continues being the *maximiser* of the good output, but units *C* and *D* generate a greater amount of bad output, while producing a lower level of good output. In other words, the downward-sloping segment of the frontier *BD* allows for the possibility that a decision-making unit can simultaneously increase production of the good output while reducing production of the bad output.

[Figure 2 about here]

In spite of the fact that economic literature on environmental performance measurement runs into hundreds of papers, few attempts have been made to explain why we might observe the downward-sloping segment of the frontier. The easiest way to justify this behaviour that apparently seems to be somewhat counter-intuitive, has been to affirm that, although observed, productive units like *D* (and also *C*) are simply outliers due to measurement error. Furthermore, despite that the theoretical framework of efficiency measurement assumes that all observations have access to the same technology, it can also be argued that points like *C* and *D* could be representing productive units using older, and also dirtier, technologies. Our perspective here is a little different since we consider that, at least, two additional reasons could justify the existence of such points.

i. On the one hand, imagine a technology generating a fixed level of bad output, irrespective of the level of production of the good output, being similar to what a fixed input is. If, in the short-term basis, a drop in the demand implies that units *C* and *D* use a poor level of their productive capacity, these units are going to be inefficient because of their sub-activity, although this sub-activity does not imply any reduction in their environmental impact.

ii. On the other hand, technology depicted in *Figure 2* could be expressive of the short-run, being the long-run technology that presented in *Figure 3*. In this new picture, we observe the extension of the efficient frontier until unit *E* (the biggest producer with the highest environmental impact), but, from the short-run perspective, we do not observe the presence of productive unit *E*.

[Figure 3 about here]

Let us now look at what would happen when we apply programs (11) and (15) to productive units *C* and *D*, using again the direction vector of expression (16), i.e., scaling only good output. Projection of points C and D onto *regulated* and *unregulated* technologies yields the following results: $\vec{D}_C^{WDB} = \vec{D}_C^{SDB} > 0$ and $\vec{D}_D^{WDB} = \vec{D}_D^{SDB} = 0$. Nevertheless, it is obvious that, for decision-making units *C* and *D*, the right benchmark is productive unit *B*, as it is demonstrated that it is possible to produce more good output with a lower level of bad output. The problem is that in the conventional approach to incorporating environmental externalities into production theory and efficiency measurement, nor the strong neither the weak disposability axioms allow identifying the existence of such situations.

In order to manage such situations, our proposal in this paper is to adapt the axiom of strong disposability of inputs to bad outputs, so that a new axiom concerning the consideration of environmental externalities as strongly disposable detrimental inputs is defined (*SDIB*):

h) *Adaptation of the axiom of strong disposability of inputs to bad outputs (SDIB)*.

$$\forall \mathbf{y}^b \in \mathbf{R}_+^B,\ if\left(\mathbf{x},\mathbf{y}^b,\mathbf{y}^g\right)\in S \Rightarrow \left(\mathbf{x},\mathbf{y}'^b,\mathbf{y}^g\right)\in S,\ \mathbf{y}'^b \geq \mathbf{y}^b \qquad (18)$$

This new axiom states that it is possible to produce the same amount of good output generating a bigger quantity of bad output also allowing for the existence of environmental inefficiency. As noted in the introduction, it might always be argued that *SDIB* violates physical laws, as strong disposal of *bads* in Färe *et al.* (1989) does. Nevertheless, in this manuscript we do not understand the *SDIB* assumption as a *real* possibility, i.e., in the *real word*, of producing unlimited quantities of contaminating wastes from given inputs, but rather as a useful empirical tool to test for the existence of situations like points *C* and *D* in *Figure 2*, i.e., where the biggest polluter is not the greater producer.

According to the assumption of *SDIB*, a new linear program can be derived to determine the *DDF* for productive unit *k* as:

$$\begin{array}{lll}
\vec{D}_k^{SDIB}\left(\mathbf{x}_k,\mathbf{y}_k^g,\mathbf{y}_k^b;-g_x,g_{y^g},-g_{y^b}\right)=Max\,\beta & & \\
\text{subject to:} & & \\
\left(\mathbf{x}_k-\beta g_x\right)\geq \mathbf{z}\,\mathbf{J} & (i) & \\
\left(\mathbf{y}_k^g+\beta g_{y^g}\right)\leq \mathbf{z}\,\mathbf{G} & (ii) & (19)\\
\left(\mathbf{y}_k^b-\beta g_{y^b}\right)\geq \mathbf{z}\,\mathbf{B} & (iii) & \\
\mathbf{ez}=1 & (iv) & \\
\mathbf{z}\geq 0 & (v) &
\end{array}$$

where, the axiom of *SDIB* has been introduced thought the inequality in restriction (19)-(*iii*).

Let us, then, return to the technology drawn in *Figure 2*, and apply program (19) to productive units *C* and *D,* orienting the direction to expand the good output while maintaining observed levels of inputs and *bads*, i.e., we continue assuming that the direction vector is that of expression (16). It is worth noting that in both cases the good output can be respectively expanded to $C''=B=C+\vec{D}_C^{DSIB}$ and $D''=B=D+\vec{D}_D^{DSIB}$. This is an information very useful for the managers of these firms, however, the regulator should also know that firms *C* and *D* are producing a level of bad output over to they would require, even in the case they maximise their production of the good output. Summing up, if there appear situations similar to productive units *C* and *D*, even without limiting the production of the good output, the regulator could achieve a effective reduction in the aggregate level of the bad output, i.e., $\left(y_C^b - y_B^b\right) + \left(y_D^b - y_B^b\right)$.

Generalising the procedure, when evaluating the efficiency of firms taking into account the environmental impact of the bad outputs, it could be possible to test for the existence of decision-making units like *C* and *D*. For a decision-making unit *k*, testing for this situation makes it necessary the definition of the following algorithm:

1. Using a direction vector that expands desirable outputs while maintaining inputs and bad outputs, i.e., the direction vector adopted is that of expression (16), compute the *DDF* $\vec{D}_k^{WDB}$, $\vec{D}_k^{SDB}$ and $\vec{D}_k^{SDIB}$, from the solution of programs (11), (15) and (19).

2. Then, compare the *DDF* computed under both *SDB* and *SDIB* axioms, that is, match distance $\vec{D}_k^{SDB}$ up with distance $\vec{D}_k^{SDIB}$.

    a. If $\vec{D}_k^{SDB} > \vec{D}_k^{SDIB}$, then the maximal increase in the good outputs requires an increase in the production of bad outputs. In this case, the impact of a regulation on the bad outputs side can be conventionally determined according to expression (17), through the comparison of *DDF* computed under *SDB* and *WDB*, i.e., distances $\vec{D}_k^{SDB}$ and $\vec{D}_k^{WDB}$.

    b. If $\vec{D}_k^{SDB} < \vec{D}_k^{SDIB}$, then the maximal increase in the good outputs reduces the level of the bad outputs. In these circumstances, the regulator can define rules to control the production of bad outputs that do not constrain the maximization of the good outputs. Consequently, $\left(y_k^g + \vec{D}_k^{SDIB}\right)$ can be the right target to achieve.

    c. Finally, if $\vec{D}_k^{SDB} = \vec{D}_k^{SDIB}$, then the maximal increase in the good outputs can be achieved with no restrictions on the bad outputs side, and $\left(y_k^g + \vec{D}_k^{SDB}\right)$ can be a target according to the environmental perspective.

In summary, our algorithm allows to identify productive units showing a particular environmental behaviour, characterised by a quite unsuccessful economic and environmental performance. Identification of these decision-making units can provide both firms managers and policy-makers with meaningful information to improve firms' environmental performance and, also, to get better designs of environmental policies. Section three illustrates our methodology applying it to a sample of Spanish ceramic tile producers.

3. Dataset and empirical illustration

In order to illustrate our methodology, we use the same dataset as in Picazo-Tadeo *et al.* (2005). The data belong to a cross-section sample of thirty five ceramic tile producers located at the region of Valencian, on the Spanish Mediterranean coast. The information comes from the *Valencian Region Inventory of Industrial Wastes* conducted in 1995 by the *Department of Environment of the Valencian Regional Government*. All firms in the sample share the same productive process, producing ceramic goods through the use of an intermediate input consisting of clay, kaolin, felspar and limestones, and two primary production factors, labour and capital. The production process also generates two undesirable products, watery mud and used oil. Output is measured in monetary units, while labour and capital factors are respectively proxied by energy consumption and the number of workers. Finally, intermediate input and bad outputs are all measured in physical units. Descriptive statistics for the data are presented in *Table 1*.

[Table 1 about here]

In order to apply our algorithm to this dataset, we have computed the *DDF* of expressions (11), (15) and (19), using the direction vector of expression (16)[7]. Under this direction, the expression for the *DDF* becomes:

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[7] Notice that this direction vector changes the left hand side of restrictions (*i*), (*ii*) and (*iii*) in expressions (11), (15) and (19) to $\mathbf{x}_k$, $\left(\mathbf{y}_k^g + \beta\right)$ and $\mathbf{y}_k^b$, respectively.

$$\vec{D}\left(\mathbf{x},\mathbf{y}^{g},\mathbf{y}^{b};0,1,0\right)=\mathrm{Sup}\ \left[\beta \middle| \left(\mathbf{x},\ \mathbf{y}^{g}+\beta,\ \mathbf{y}^{b}\right)\in S\right] \tag{20}$$

This distance measures the extent to which a firm could increase its production of good output, while maintaining inputs and *bads*. In other words, given their endowment of inputs, it could be possible for *inefficient* tile firms to move to cleaner environmental-friendly productive plans that would enable them to produce more ceramic pavements while holding the same amount of watery mud and used oil. Since the amount of ceramic pavements increases with constant amount of *bads*, the ratio of each bad output per unit of good output will decrease.

Comparing the *DDF* computed under the assumptions of *SDB* and *SDIB*, i.e., the solutions to programs (11) and (15), shows that 29 firms (out of 35) present a behaviour corresponding to the *basic situation* depicted in *Figure 1* (see *Table 2*). For these firms, the maximal increase in their production of ceramic pavements, requires an increase in the production of watery mud and used oil. When no regulation is assumed and wastes can be freely disposed, the aggregate desirable output produced by these firms could be augmented by 25 per cent, figure that amounts to 64,915 thousands of euros, with an average of 2,238 thousands per firm. Besides, eleven (out of 29) firms behave efficiently in this *unregulated* scenario, i.e., their computed *DDF* equals to zero. In opposition, when it is assumed a *regulated* scenario and disposing wastes becomes a costly activity, the maximum attainable aggregate expansion of *goods* goes down to 16,982 thousands of euros (537 euros per firm), showing that environmental regulations have an opportunity cost measurable in terms of a lower feasible expansion of desirable outputs. In this case, the reduced production of ceramic pavements due to inefficiency represents 6.5 per cent of the total good output produced by these firms, and twenty two tile firms show an efficient behaviour.

[Table 2 about here]

For these ceramic firms behaving according what we have termed the *basic situation*, the impact of environmental regulation can be conventionally computed as stated by expression (17), i.e., as the difference between the potential increases of ceramic pavements under both the *unregulated* and *regulated* technologies. Figures on regulatory impacts are reported on the third column of *Table 2*. When it is assumed that environmental rules prevent free disposal of *bads*, in aggregate terms these producers would have to renounce to a potential increase of desirable output of 47,933 thousands of euros (1,635 per firm). This figure amounts to 18.5 per cent of their observed aggregated production of ceramic pavements.

Furthermore, comparison of *DDF* computed under the *SDB* and *SDIB* axioms also reveals that for the six remaining firms (decision-making units 1, 7, 11, 13, 24 and 27) increasing production of good reduces the level of the bad outputs, i.e., they are productive units projected on a downwards-sloping piecewise of the frontier. In particular, firm number 13 displays a behaviour analogous to productive unit *C* in *Figure 2*, while firms 1, 7, 11, 24 and 27 are productive units similar to point *D*. The simple comparison of *DDF* computed under both *SDB* and *WDB*, would lead to assess that, given their respective inputs vectors, no chance exists for the five firms type *D* to attain increases of their production of ceramic pavements, neither under the *unregulated* scenario nor under the *regulated* one. On the contrary, by behaving efficiently, the only firm type *C* in our sample (productive unit number 13) could increase its production of ceramic pavements by 1,821 thousands of euros, under both *unregulated* and *regulated* technologies. Accordingly, environmental regulation is not binding for productive units type *C* and *D*, and their regulatory impact index equals to zero.

Nevertheless, considering the axiom of *SDIB* displays a picture somewhat different for these six tile firms. In the light of *DDF* computed under *SDIB*, these decision-making units could attain an aggregate increase in their production of ceramic pavements of 13,026 thousands of euros, without additional consumption of inputs, with an average per firm of 2,171 thousands of euros. Doubtless, this is an information of great interest for the managers of these firms. Even so, regulating authorities should also be aware that these tile firms are producing a level of bad output over to they would require, even in the case they maximize their production of ceramic pavements. In this situation, an effective reduction in the aggregate level of watery mud and used oil could be achieved by environmental-friendly regulations, without limiting the production of the good output.

4. Concluding remarks

Environmental performance is a major matter of concern for both firms' managers and academics in the field of environmental economics. Besides, the growing recognition of the environment as a public good in the industrialised countries has stimulated wide-ranging legislation aimed at achieving predetermined standards of environmental quality. This setting calls for methods to evaluate the environmental performance of firms and the impact of environmental-friendly regulations on their productive activity.

From the pioneering manuscript by Färe *et al.* (1898), environmental externalities that frequently generate production processes have been considered as undesirable outputs under either weak or strong disposability. Treating polluting wastes as outputs has revealed itself as a powerful empirical tool to incorporate environmental externalities into traditional production theory and efficiency analysis. However, our argument in this paper is that considering environmental externalities as strongly detrimental inputs constitutes also a sensible empirical tool that helps to identify technologies where the biggest producer is not the greater polluter, i.e., technologies located on the upward-sloping segment of the frontier depicted in Färe *et al.* (1989). Moreover, our paper provides economic arguments supporting the possibility that these observations exist, going beyond the conventional claims that consider that they could correspond to outliers or measurement errors.

By using directional distance functions, we define an algorithm that allows to unambiguously identify such type of production technologies. In addition, we show that when the biggest polluter is not the greater producer, by enforcing adequate environmental regulations, public authorities could achieve an effective reduction on the production of *bads*, without affecting the level of *goods*. Furthermore, we illustrate our methodology with an empirical application to a sample of cross-section data of Spanish tile producers.

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Table 1.- Sample description.

Variable	*Measurement unit*	*Mean*	*Standard deviation*
Desirable output			
Ceramic pavements	Euros (thousands)	8,814.1	11,470.0
Undesirable outputs			
Watery mud	Tons	1,725.9	3,411.6
Used oil	Kilograms	1,537.4	2,460.5
Inputs			
Clay, kaolin, felspar and limestones	Tons	58,182.3	102,559.3
Labour	Number of workers	87.0	101.5
Capital	Kilowatts-hour (thousands)	3,408.4	4,006.8

Table 2.- Computed distance functions, regulatory impacts and type of firms.

	Bads as undesirable outputs			*Bads as strong disposable inputs*	
Firm	*Unregulated scenario (SDB)*	*Regulated scenario (WDB)*	*Regulatory impact index*	*SDIB*	*Firm type*
1	0	0	0	1,151	*D*
2	0	0	0	0	-
3	3,918	0	3,918	0	-
4	2,624	1,208	1,416	1,208	-
5	0	0	0	0	-
6	1,113	184	929	228	-
7	0	0	0	2,274	*D*
8	16,274	11,551	4,723	11,551	-
9	0	0	0	0	-
10	0	0	0	0	-
11	0	0	0	3,171	*D*
12	0	0	0	0	-
13	1,821	1,821	0	4,269	*C*
14	0	0	0	0	-
15	433	0	433	0	-
16	12,215	0	12,215	0	-
17	0	0	0	0	-
18	2,693	0	2,693	0	-
19	1,327	0	1,327	0	-
20	1,149	0	1,149	535	-
21	7,270	539	6,731	759	-
22	2,011	700	1,311	588	-
23	0	0	0	0	-
24	0	0	0	1,479	*D*
25	0	0	0	0	-
26	1,449	0	1,449	0	-
27	0	0	0	682	*D*
28	336	0	336	0	-
29	0	0	0	0	-
30	3,241	0	3,241	0	-
31	2,461	0	2,461	375	-
32	596	0	596	0	-
33	3,703	1,880	1,823	1,999	-
34	2,102	920	1,182	920	-
35	0	0	0	0	-

Figure 1.- Weak disposability of outputs (I): the *basic situation*.

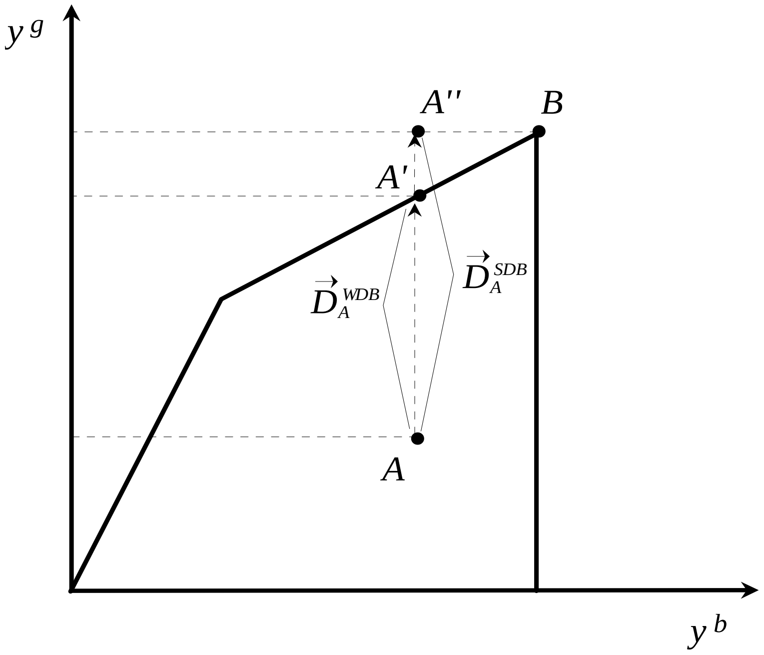

Figure 2.- Weak disposability of outputs (II): the *downward-sloping segment*.

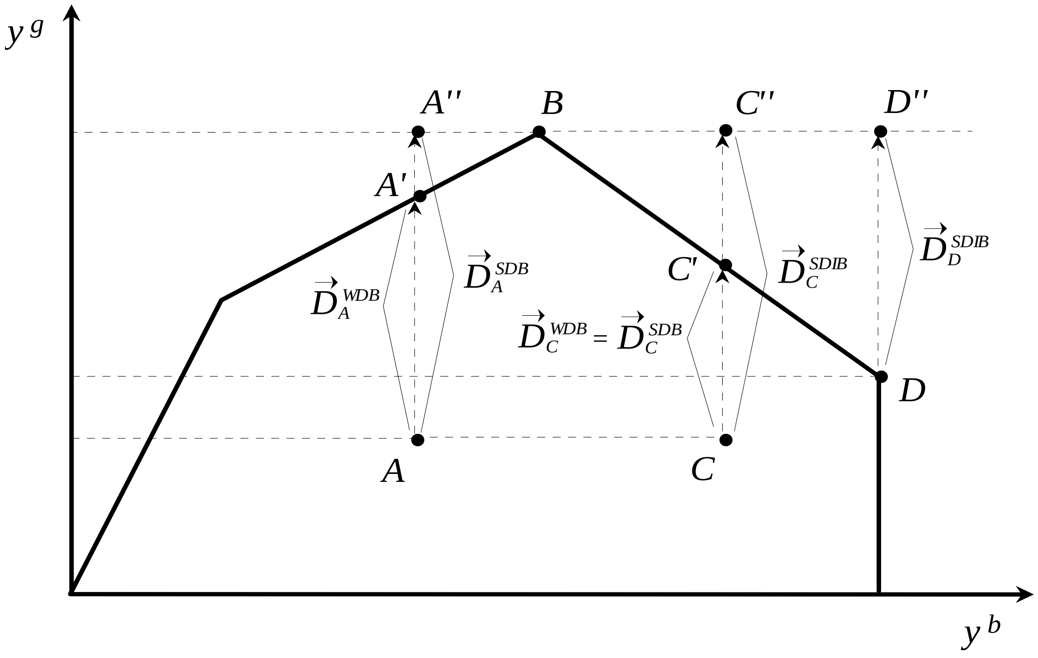

Figure 3.- Weak disposability of outputs (III): the *long-run perspective*.