

# Cognitive Consistency, the Endowment Effect and The Preference Reversal Phenomenon

By SERGE BLONDEL AND LOUIS LÉVY-GARBOUA \*

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*We exhibit three non-standard cases of preference reversal (PR) between choices and valuations of bets. The endowment effect arises as a surprising consequence of the WTA/WTP disparity when the probabilities of winning get close to certainty. We consider a new theory of choice under risk, cognitive consistency (CC) theory, which can predict both standard and non-standard cases of PR, the WTA/WTP disparity and the endowment effect in the context of single choices. The experimental results strongly support CC predictions with a single parameter added to risk aversion, reject the regret theory explanation for PR and the loss aversion explanation for the endowment effect. (JEL C91, D81)*

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- Blondel ([serge.blondel@inh.fr](mailto:serge.blondel@inh.fr)): National Institute of Horticulture (INH), Angers and Centre d'économie de la Sorbonne, Université de Paris I. Lévy-Garboua ([louis.levy-garboua@univ-paris1.fr](mailto:louis.levy-garboua@univ-paris1.fr)): Centre d'économie de la Sorbonne, Université de Paris I. We are grateful to Enrico Decidue, Stephane Robin and Gerritt Roth for careful reading and very helpful comments, to Antoine Terracol for helpful advices, and to Ronan Symoneaux for his help in programming and conducting the experiment. These experiments were financed by GIS "Cognition" and INH. We remain responsible for any errors.

The preference reversal phenomenon (from now on, PR), first reported by psychologists (Sarah Lichtenstein and Paul Slovic, 1971, Harold Lindman, 1971) can be described by the following example. Individuals are confronted with two lotteries with close expected gains, often called “P-bet” (a large probability of winning a small prize) and “\$-bet” (a small chance of winning a large prize). Subjects are asked to indicate their preference between the two lotteries, either by choosing one of the two lotteries or by placing a monetary value on each of the lotteries. The order in which these two tasks are accomplished is indifferent.

For instance, in one typical experiment, the P-bet yielded a gain of 6€ (1€ is worth about \$1.20) when subjects drew a number between 1 and 18, and 0 if the number lay between 19 and 20. The \$-bet offered a prize of 20€ when subjects drew a number between 1 and 6, and 0 if the number lay between 7 and 20. The standard PR phenomenon was replicated in this experiment: amongst the subjects who chose the P-bet, 82 percent of set a higher minimum selling price on the \$-bet (for an extensive survey on PR, see Christian Seidl, 2002).

The PR phenomenon is a major challenge to the economic theory of choice (David Grether and Charles Plott, 1979) because it demonstrates that the revealed preference depends upon the elicitation procedure which is being used. Two normatively equivalent procedures (choosing and pricing) may elicit opposite preferences. Such result violates the postulate of procedure invariance which is common to normative theories of choice under risk like expected utility (EU) theory, rank-dependent expected utility or cumulative prospect theory.<sup>1</sup>

An explicit assumption of framing is needed to overcome this major paradox. The psychological theory of contingent weighting<sup>2</sup> (Amos Tversky et al., 1988) and the economic theory of regret (Graham Loomes and Robert Sugden, 1983) both make explicit assumptions

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<sup>1</sup> We define as “normative” any theory of choice that defines preference as being presentation and procedure-invariant, or context-free.

<sup>2</sup> Expression theory (William Goldstein and Hillel Einhorn, 1987) is a special case of contingent weighting.

of framing and are consistent with cases of PR examined in the earlier literature. Contingent weighting theory elaborates the initial intuition of Slovic and Lichtenstein (1968) and attributes PR to prominence effect and compatibility scale. In a choice task, the probability of winning is a more prominent attribute of lotteries than outcomes because individuals are not sure of winning, and thus weighs more heavily in the preference index. This favors the choice of  $P$  over  $\$$ . In a pricing task, the winning outcomes become prominent because they measure on the same scale as prices. This now leads to a higher value of  $\$$  than  $P$ . Since the more prominent attribute changes with the task used to elicit preferences, procedure invariance does not hold and PR may be observed. Regret theory too may accommodate PR as a result of postulating a reference-dependent utility function in which the reference depends on the preference elicitation procedure. The natural reference for  $\$$  is lottery  $P$  in a choice task, but it is a sure amount,  $\$$ 's reservation price, in a valuation task. Although the reference-dependent utility function used by regret theory may generate an intransitivity of preferences, the primary reason for causing PR is violation of procedure invariance (Tversky et al. 1990).

However, we exhibit nonstandard cases of PR which are inconsistent with these two models. First of all, previous experiments have assumed that PR requires two lotteries but we present evidence that PR can also occur when a single lottery is confronted with a sure outcome. This new form of PR, designated as PR1, is inconsistent with regret theory. In addition, both contingent weighting and regret theory predict the same cases of PR, whether prices are determined as a minimum selling price or as a maximum buying price, when the income effect can be neglected. However, we find a very different type of PR when maximum

buying prices are being used rather than minimum selling prices to determine lottery values<sup>3</sup>. Lastly, PR does not require small probabilities of winning for the riskier bet.

Our paper considers a new theory of decision under risk, called cognitive consistency theory (Louis Lévy-Garboua, 1999), which makes an explicit assumption of framing. Cognitive consistency (henceforth, CC) is a theory of the decision process based on the sequential perception of two dissonant cognitions: the prior preference (EU in the context of single choices) and objection to the latter. CC theory has several nice features for our study. First, it is a true generalization of EU as it can predict when the prior EU preference will be either revealed or violated. Second, it yields for pairwise choices and values a simple and parsimonious criterion. The latter can be described by an objection-dependent expected utility (ODEU) with a single parameter, the objection's weight, in addition to risk aversion. Third, it shows that the BDM (Gordon Becker, Morris DeGroot, Jacob Marshak 1964) mechanism and the random lottery incentive system are incentive-compatible. Thus, we can study the willingness to accept (WTA), willingness to pay (WTP), WTA/WTP disparity and PR with these widely used methods on the same data set. . The conception of our experiments derives from implications of CC theory which partly contradict those of regret theory and contingent weighting theory.

Jack Knetsch and John Sinden (1984) first reported for lotteries that WTA was significantly greater than WTP, well above what income effects alone can explain. However, the WTA/WTP disparity has also been observed with “seemingly riskless” goods<sup>4</sup>, like a

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<sup>3</sup> Jeff Casey (1991, 1994) had already found cases of reverse PR by using a maximum buying price to determine the lottery values, and Ulrich Schmidt and John Hey (2004) make an explicit use of buying prices to refute contingent weighting theory.

<sup>4</sup> The expression is from Lévy-Garboua and Claude Montmarquette (1996), for reasons which will become apparent later on.

coffee mug and a bar of chocolate. A related phenomenon, typically observed on the same kind of “seemingly riskless” goods, is the endowment effect, that is, the fact of giving a higher value to a good if one owns it than if one doesn’t own it (Richard Thaler, 1980). The main interpretation which has been given for the endowment effect and the WTA/WTP disparity in the literature is “loss aversion” (Tversky and Daniel Kahneman, 1991), that is, the fact that people seem to attribute a substantially higher absolute value to a loss than to an equal gain. Owners of a good (lottery) would be more reluctant to lose the latter by selling it than non owners would be keen to enjoy this good by purchasing it. Since the endowment effect on “seemingly riskless” goods also applies to beneficial bets, it is tempting to relate it with the WTA/WTP disparity for lotteries by varying the win-probabilities of true bets from 0.10 to 0.95. Obviously, people who own a good (lottery) tend to value it by a descending auction, as a minimum selling price, while people who don’t own this good (lottery) tend to value it by an ascending auction, as a maximum buying price. What we observed came to us as a surprise but was actually predicted by CC theory. Asking prices of bets converged to the maximum gain when the probabilities of winning were getting close to one, but bidding prices remained significantly below this value. Thus, the endowment effect does not arise for “seemingly riskless” goods from an overestimation of goods owned but from an underestimation of goods not owned. This finding contradicts the loss aversion explanation of the endowment effect.

In order to show that CC can predict both standard and nonstandard PR, an outline of this new theory is provided in section I in the context of PR experiments, i.e. choosing under risk and pricing bets. Theoretical predictions on the PR phenomenon, the WTA/WTP disparity and the endowment effect are then derived from this model in section II for single tasks. These predictions have motivated a new experimental design which is summarized in section III. We show in particular that the commonly used random lottery incentive system and the

BDM mechanism are incentive compatible if subjects satisfy CC theory. The results of our experiment are then presented and discussed in section IV. We conclude in section V.

## I. Cognitive Consistency theory

### A. Choosing under risk

In all the experiments, we consider single choices between  $P$  and  $\$$  bets of the form  $(y, r; 0, 1-r) \equiv (y, r)$  with  $y > 0$ ,  $0 < r \leq 1$ , which offer gain  $y$  with the stated probability  $r$  and 0 otherwise. We write  $\$ = (y_\$, r_\$)$  and  $P = (y_P, r_P)$ , with  $y_\$ > y_P$  and  $r_\$ < r_P \leq 1$ . Thus the P-bet may be a sure gain but the  $\$$ -bet is always risky. Since we consider three outcomes at most, we simplify notations by normalizing the utility of wealth in the following manner:  $U(w+y) \equiv u(y)$ , where  $w$  designates initial wealth and  $y$  any monetary outcome;  $u(y_\$) = 1$ ,  $u(0) = 0$ ,  $u(y_P) = x$ ,  $0 < x < 1$ . Consequently,  $EU(\$) = r_\$$  and  $EU(P) = r_P x$ .

When the actual outcome of the preferred lottery is to be determined by a roulette wheel or the like, lotteries are naturally framed as *actions* with well-specified states of the world. Table 1 represents the choice between the P-bet and the  $\$$ -bet under the assumption that  $P$ , as well as  $\$$ , is a nondegenerate bet (i.e.  $r_P < 1$ ).

[Table 1 here]

Let us consider that, prior to making a choice, the individual has a normative, i.e. procedure-invariant, preference under risk which can be represented by an EU function. Whether this prior preference be  $P$  or  $\$$ , it raises *doubt* when the subsequent choice of one lottery against another raises a visible *objection* (see table 1). For instance, if  $\$$  is EU-preferred to  $P$ , the visible objection to choosing  $\$$  is that the most unfavourable state might occur, with probability  $r_P - r_\$$ , and leave the individual with 0 outcome instead of the strictly greater sum

$y_p$  if she had opted for  $P$ . Conversely, if  $P$  is EU-preferred to  $\$,$  the visible objection to choosing  $P$  is that the highest outcome  $y_s$  might have been won instead of  $y_p,$  with probability  $r_s,$  by opting for  $\$.$  The possibility of finding an objection to one's normative preference, which characterizes most decisions under risk or uncertainty, means that the decision-maker demands *information.* In seeking additional information, she must perceive the available objection to her normative preference. Thus she must *sequentially* perceive, first her normative preference, then the available objection to the latter. In the first (second) example listed above, after perceiving her EU-preferred lottery  $\$ (P)$  as the risky action described in table 1, she will perceive it quite differently as if the unfavourable (most favourable) state of the world represented in the second (first) column of table 1 occurred with certainty. The latter, called the "objection state", is conditional on the normative preference. Since the objection is always dissonant with the prior preference, the individual experiences cognitive dissonance and must feel uncertain of her true preference (for further developments, see Lévy-Garboua and Serge Blondel, 2002, Lévy-Garboua, 2004). More precisely, the decision-maker observes two values of the objection's likelihood, first the objective probability of the objection state  $p$  (i.e.,  $r_p - r_s$  in the first example and  $r_s$  in the second), then probability 1.

In a pairwise comparison, the *unknown preference* of a rational individual can be treated as a Bernoulli variable with an unknown value of the parameter in the unit interval, the objection's likelihood  $\tilde{p}.$  It is assumed that observed values of this random variable are independent identically distributed. A Bayesian subject unsure of her true preference will search for *cognitive consistency* by making use of all available information, and attach a positive weight to the objection. If she has no experience or prior information about the gambles, she makes a single choice, and if the prior distribution of  $\tilde{p}$  is Beta with two

positive parameters  $(\varphi, \psi)$ , then the given prior mean is  $p = \frac{\varphi}{\varphi + \psi}$  and the posterior mean (see DeGroot 1970: chapter 9, for instance):

$$\begin{aligned} p^* &= \frac{\varphi + 1}{\varphi + \psi + 1} \\ &\equiv \frac{\varphi + \psi}{\varphi + \psi + 1} p + \frac{1}{\varphi + \psi + 1} 1 \end{aligned}$$

The posterior mean is the *perceived* likelihood of the objection. It is a weighted average of the two cognitions, the weights being proportional to the respective precisions of the normative value and the objection. Let us designate these weights as the “prior’s weight” ( $\mu > 0$ ) and the “objection’s weight” ( $1 - \mu$ ). Thus we write equivalently with these notations:

$$\begin{aligned} (1) \quad p^* &= \mu p + 1 - \mu \\ &= p + (1 - \mu)(1 - p) \end{aligned}$$

The last formulation shows that the perceived objection’s likelihood exceeds the objective probability whenever individuals are not perfectly sure of their prior preference. The smaller is the objective probability of the objection state, the more is the perceived probability being over-weighted. *CC theory yields a simple theory of probability transformation by postulating that the prior’s weight is an individual-specific parameter for a class of related decision problems.* It predicts rational deviations from the normative preference (EU) caused by sequential perception, since the latter induces cognitive dissonance and preference uncertainty. It systematically boils down to EU if  $\mu = 1$ . In choosing between two risky bets  $(B_1, B_2)$ , rational individuals who feel uncertain of their normative preference for  $B_1$ , instead of maximizing EU, will maximize *objection-dependent expected utility* (ODEU) conditional on their normative preference:

$$(2) \quad ODEU(B_k / B_1) = \mu EU(B_k) + (1 - \mu) EU(b_k / B_1), \quad k = (1, 2),$$

in which the ODEU of lottery  $B_k$  conditional on EU-preference for  $B_1$  is denoted as  $ODEU(B_k / B_1)$  and  $b_k$  is the outcome (a sure outcome or a lottery) of  $B_k$  in the objection state conditional on the normative preference. The subject would be sure of her normative preference when  $\mu$  equals 1 (EU-maximizer) and almost sure of the objection when  $\mu$  is close to 0, but she would normally lie in-between. It is worth noticing that ODEU is a reference-dependent rule in which the EU preferred action  $B_1$  is a natural reference.

**PROPOSITION 1** (ODEU rule for pairwise choices between risky actions)

*A rational individual whose normative preference under risk is EU maximizes her objection-dependent EU (ODEU) conditional on her EU-preferred action when making a single decision between two risky actions. Then the ODEU is a subjective EU in which the perceived likelihood of the objection's state (conditional on the EU-preference) exceeds the objective probability when the individual has no experience or prior information about the gambles.*

For instance, with subjects EU-preferring \$:

$$(3) \quad \begin{aligned} ODEU(\$ / \$) &= \mu EU(\$) + (1 - \mu)0 \\ ODEU(P / \$) &= \mu EU(P) + (1 - \mu)x \end{aligned}$$

And, with subjects EU-preferring P:

$$(4) \quad \begin{aligned} ODEU(\$ / P) &= \mu EU(\$) + (1 - \mu)1 \\ ODEU(P / P) &= \mu EU(P) + (1 - \mu)x \end{aligned}$$

Since EU-preference and the revealed preference may either coincide or differ, the sequence of perceived preferences for either  $P$  or  $\$$  can follow four *decision paths* in an action frame. Using expressions (3) and (4), these are summarized by **PP**, **P\\$**, **\\$P**, **\\$\\$**, in which the first symbol indicates the prior preference (EU) and the second symbol the posterior preference (ODEU).

When  $\$$  is the prior EU-preference<sup>5</sup> (*i.e.*  $EU(P) < EU(\$)$ ), (3) shows that the posterior ODEU-preference will be  $P$  if and only if  $\mu EU(P) + (1 - \mu)x > \mu EU(\$)$ , which implies  $\mu < 1$ . Conversely, when  $P$  is the prior EU-preference (*i.e.*  $EU(P) > EU(\$)$ ), the posterior ODEU-preference will be  $\$$  if and only if  $\mu EU(\$) + (1 - \mu)1 > \mu EU(P) + (1 - \mu)x$ , which also implies  $\mu < 1$ . Notice that the *ODEU* function is not continuous at the point of indifference (*i.e.*  $r_s = r_px$ ), as a result of the change in the prior and objection state across this point.

## B. Pricing bets

In CC theory, it is not equivalent to evaluate a gamble by descending auctions (minimum selling price) or by ascending auctions (maximum bidding price). This can explain the observed WTA/WTP disparity and the endowment effect. It can also predict different types of PR whether lotteries are valued in one way or another. Let  $p_a$  denote the asking or minimum selling price, and  $p_b$  the buying or maximum bidding price of a nondegenerate gamble.

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<sup>5</sup> If  $P$  and  $\$$  are EU-indifferent, the prior preference cannot be selected by the EU criterion alone and will be determined by an arbitrary criterion. CC theory predicts in this special case that the final choice will systematically violate this arbitrary prior. However, insofar the criterion used for selecting the prior is unknown to the observer, EU-indifference transposes into CC-indifference.

The owner of a bet  $B = (y, r)$  compares the utility of playing this gamble with the utility she would draw from selling this lottery and getting a sure amount of money in exchange. In a descending auction, she starts with a high price (say, the maximum gain  $y$ ) and lowers the price until she is indifferent between selling her rights to gamble and playing the gamble herself. Thus it can be safely assumed that the prior preference is to add a high money price to endowed wealth  $w$ :  $U(w + p_a) > EU(w + B)$ . Hence, the objection to selling is that the price is still lower than the maximum gain from playing, and

$$(5) \quad ODEU(B / p_a) = \mu EU(w + B) + (1 - \mu)U(w + y)$$

The asking price is such that the two options are ODEU-indifferent:

$$(6) \quad \mu EU(w + B) + (1 - \mu)U(w + y) = U(w + p_a)$$

The utility of selling the lottery is a weighted average of the EU of playing the bet and the utility of winning the prize. The WTA for a nondegenerate bet is always greater than its EU-certainty-equivalent for subjects who are uncertain of their normative preference. CC theory confirms the initial intuition of Slovic and Lichtenstein (1968) when the value of the bet is determined by a descending auction: subjects reporting their asking price focus on the maximum gain. The prominence effect is exacerbated when the objection is heavily weighted.

By contrast, the individual who considers buying the right to play the same bet must compare the utility of her own wealth  $w$  with the utility drawn from paying a price and then playing the gamble. In an ascending auction, she starts with a low price (say, the minimum gain 0) and increases her bid until she becomes indifferent between the two options. Now the prior preference is to buy the bet at a low price:  $EU(w - p_b + B) > U(w)$ , and the objection to buying is that the net wealth after playing the gamble might be lower than the sure initial wealth. Hence,

$$(7) \quad ODEU(B/B) = \mu EU(w - p_b + B) + (1 - \mu)U(w - p_b)$$

The buying price is such that the two options are ODEU-indifferent:

$$(8) \quad \mu EU(w - p_b + B) + (1 - \mu)U(w - p_b) = U(w)$$

Let  $CE(B)$  designate the EU-certainty equivalent of bet  $B$ . Since (8) and  $U(w) = 0$  imply  $EU(w - p_b + B) > 0$  if  $\mu < 1$ , and  $EU(w - CE(B) + B) = 0$ , we must have:  $p_b < CE(B)$  for individuals who are unsure of their normative preference. Putting the opposite implications of (6) and (8) together, CC theory predicts the *WTA/WTP disparity* for nondegenerate bets (Knetsch and Sinden, 1984):

$$(9) \quad p_b < CE(B) < p_a \quad \text{for all } \mu < 1$$

The WTA/WTP disparity implies underweighting of the maximum gain in ascending auctions and overweighting in descending auctions. Although the objection state is always overweighted, its very definition depends upon whether the price is determined by a descending or an ascending auction. The objection to selling a bet at a competitive price is the possibility of winning if one played it, while the objection to buying a bet at a competitive price is the possibility of losing. CC theory posits that the WTA/WTP disparity on lotteries is due to the fact that individuals, out of self-interest, tend to rely on descending auctions to determine their minimum price for a lottery ticket which they have the right to sell, and on ascending auctions to determine their maximum buying price. As a corollary, individuals tend to evaluate a lottery by a descending or an ascending process indifferently when they are not urged explicitly to sell or to buy. In the latter case, the *average* value of the lottery should be equal to:  $(P_a + P_b)/2$ .

## II. The preference reversal phenomenon, WTA/WTP disparity and the endowment effect

In this section, we study CC theory's predictions of PR, assuming that choices are made in an action frame, which is typically the case in PR experiments. We demonstrate that different types of PR can be observed with asking prices and with buying prices. Then PR is extended to a single lottery. The WTA/WTP disparity and a new explanation for the endowment effect emerge from this analysis.

*A. PR with asking prices and with bidding prices*

Standard PR is neatly predicted by CC theory if preferences are elicited from asking prices (see table 3a), and non standard PR is predicted if preferences are elicited from bidding prices (see table 3b). While standard PR is asymmetric and restricted to subjects choosing the  $P$  (less risky) bet, the non standard PR is symmetric and extends to subjects choosing either the  $P$  bet or the  $\$$  bet. The theoretical predictions are described in more detail by proposition 3 (proofs shown in appendix).

**PROPOSITION 2** (necessary and sufficient conditions for PR)

*Assume that the preference between a  $P$  bet and a  $\$$  bet satisfies CC theory and that none of these bets is degenerate. If these preferences are elicited independently in a single task context by a choice between  $P$  and  $\$$  and by a separate evaluation of  $P$  and  $\$$ , then the PR phenomenon (PR) occurs under the following conditions:*

*(a) If bets are both evaluated by their minimum selling price (asking price), following the  $\$P$  decision path is a necessary and sufficient condition for PR.*

*(b) If bets are both evaluated by their maximum buying price (bidding price), and if EU-preferences are wealth-neutral (i.e. unaffected by variations of wealth), following either  $\$P$  or  $P\$$  decision paths is a necessary and sufficient condition for PR.*

[Tables 2a and 2b here]

In the selling condition, CC predicts that PR essentially occurs when subjects choose the P-bet and give a higher value to the \$-bet. CC's prediction is even more specific: PR is restricted to subjects who followed the **\$P** decision path, that is, who chose the P-bet after EU-preferring the \$-bet. A simple way of testing these predictions consists of comparing bets with small or moderate gains and (not too) different expected values. Hence, the EU-preference can be characterized, for a large majority of subjects, as being the bet with a higher expected gain irrespective of subjects' risk attitude (Matthew Rabin, 2000). This methodology will be adopted in our experiments.

CC also predicts PR in the buying condition. However, PR is now symmetric as it may occur for subjects who deviated from their normative preference irrespective of the latter. This will be true for risk-neutral subjects and, more generally, for wealth-neutral subjects, *i.e.* if EU-preference remains the same at all levels of wealth. Thus individuals who chose the \$-bet are concerned with PR as much as those who chose the P-bet, and a non-standard or reverse PR can now also be observed for wealth-neutral subjects. These predictions of CC theory diverge from those of contingent weighting theory and regret theory but they are in line with previous findings of Casey (1991, 1994).

#### B. PR with a single lottery

The analysis of choice is not modified under CC when the P-bet degenerates into a sure gain because the objection states remain unaffected, irrespective of EU-preference, when the last column is suppressed in table 1 (*i.e.*  $r_p = 1$ ). Therefore, the two PR phenomena described in proposition 3 would readily extend to a single lottery by continuity if the two pricing equations were continuous around certainty (*i.e.* if  $p_i(P) \rightarrow y_p$  as  $r_p \rightarrow 1$  for  $i = (a, b)$ ). However, this happens to be true for asking prices only as can easily be seen from (10). When values are elicited by asking prices, PR should occur with a single lottery (PR1) in the way it occurred with two lotteries. This is a novel prediction of CC which is inconsistent with regret

theory (Blondel and Lévy-Garboua, 2006). However, PR1 sharply differs from PR when values are elicited by bidding prices since the buying price equation (10) is not continuous around certainty when  $\mu < 1$ : if  $r_p \rightarrow 1$ ,  $p_b(P) \rightarrow \mu y_p < y_p$ .

### **PROPOSITION 3 (PR1)**

*Assume that the preference between a sure gain  $y_p$  and a nondegenerate \$ bet satisfies CC theory. If these preferences are elicited independently in a single task context by a choice between P and \$ and by a separate evaluation of \$, then the PR1 phenomenon occurs under the following conditions:*

*(a) If the \$ bet is evaluated by its minimum selling price (asking price), following the **\$P** decision path is a necessary and sufficient condition for PR1. This result replicates PR in the selling price condition.*

*(b) If the \$ bet is evaluated by its maximum buying price (bidding price), PR1 is essentially restricted, under common experimental conditions (i.e. P and \$ yield small gains and have close expected values), to subjects unsure of their prior preference ( $\mu < 1$ ) and choosing \$.*

[Tables 3a and 3b about here.]

Tables 3a and 3b summarize predicted PR1 when values are respectively captured by asking prices and bidding prices. PR1 replicates PR in the selling price condition. However, a new pattern of PR arises in the buying price condition since PR1 is now essentially restricted to subjects who chose the \$-bet over a sure gain but indicate a buying price for the lottery which is lower than the sure gain (proofs shown in appendix). It should be emphasized that, with the exception of the **PP** decision path, the described pattern of PR1 in the bidding price condition (table 3b) is an approximation of reality. The level of approximation, though, should be acceptable under common experimental conditions. If gains are small relative to initial wealth

and if  $P$  and  $\$$  have close yet unequal expected gains, the assumption of risk neutrality generates a small error which would be dominated by the effect of a substantial objection's weight. More precise approximation formulae will be given in section 4 (equation (12)). However, the conditions under which the approximation is valid reflect more fundamentally the natural boundaries of the PR phenomenon. Indeed, CC theory predicts that no PR phenomenon of any type will ever be observed if subjects have "strong" preferences for either  $P$  or  $\$$ , in the sense that the  $EU$  gap is large and straight decision paths **PP** or **\$\$** are followed. Undeniably, these conclusions have an intuitive appeal and demonstrate that the so-called "experimental" conditions are typically those needed to make the PR and PR1 phenomena observable.

### C. WTA/WTP disparity

Since the asking price and the buying price are reported by subjects, we solve equations (6) and (8) for these two variables as a function of the maximum gain of the bet and the two risk premia  $\Pi_a(B)$  and  $\Pi_b(B)$  (whose signs follow the sign of  $-U''$ ):

$$(10) \quad p_a(B) = (1 - \mu(1 - r))y - \Pi_a(B); \quad p_b(B) = \mu r y - \Pi_b(B)$$

The magnitude of the WTA/WTP disparity is given by:

$$(11) \quad p_a(B) - p_b(B) = (1 - \mu)y + \Pi_b(B) - \Pi_a(B)$$

Under the assumption of risk neutrality, the risk premia vanish and the WTA/WTP disparity takes a remarkably simple expression. Then the ratio of the WTA/WTP disparity to the maximum gain equals the objection's weight which does not depend on the lottery's description.

### **PROPOSITION 4** (WTA/WTP disparity)

*Assume that people evaluate the minimum selling price of a lottery by a descending auction and its maximum buying price by an ascending auction. Then, under CC theory, the minimum*

*selling price is higher than the maximum buying price of a beneficial bet if  $\mu < 1$ , after controlling for the income effect. For risk-neutral individuals, the ratio of the WTA/WTP gap to the maximum gain equals the objection's weight  $1 - \mu$ , which does not depend on the characteristics of the bet.*

#### D. *The endowment effect*

The discontinuity of the relation of buying prices of bets with their probability of winning close to certainty and the continuity of asking prices can also predict the *endowment effect*. The endowment effect is the fact of giving a higher value to a good if one owns it than if one doesn't own it (Thaler, 1980). Since this effect also applies to beneficial bets (Knetsch and Sinden, 1984), it is natural to relate it with the WTA/WTP disparity for lotteries. Obviously, people who own a good (lottery) tend to value it by a descending auction, as a minimum selling price, while people who don't own this good (lottery) tend to value it by an ascending auction, as a maximum buying price. Moreover, potential sellers of a good should detain idiosyncratic information about that good which potential buyers don't have. This information could even be induced by the pleasure of receiving a small gift in an experiment (Lévy-Garboua and Montmarquette, 1996). Therefore, non-owners of a good should perceive a "small" risk of failure and treat that good as a lottery, while owners feel pretty much certain of the good's quality. Due to the presence of a small perceived risk of failure and the discontinuity of buying prices of lotteries around certainty, non-owners of a good (lottery) will value the latter markedly below its true value under certainty. This effect would disappear if the good (lottery) was replaced by induced-value tokens (Kahneman et al., 1990) as the exchange of tokens with cash bears no uncertainty. Finally, CC theory posits that values of goods (lotteries) are not overestimated by owners but underestimated by non-owners.

**PROPOSITION 5** (the endowment effect)

*Assume that the preference for nondegenerate bets offering a positive probability to win a gain  $y$  satisfies CC theory and that  $\mu < 1$ . When the probability of winning approaches 1, the asking price converges toward  $y$ , but the bidding price reaches a lower value  $\mu y$ . If “seemingly riskless” goods bear a small uncertainty as long as one does not own them, an endowment effect is created by the underestimation of the goods’ value by non-owners.*

Proposition 5 offers an alternative to the “loss aversion” explanation for the endowment effect which has the merit of being testable. The design of a new experiment will achieve this goal.

**III. Experimental design**

Sixty-two subjects were recruited among the students of a college-level school of agronomy (Institut National d’Horticulture – INH - or Ecole Supérieure d’Agriculture - ESA) at Angers (France). The experiments were run in May 2004, at the ESA laboratory for sensorial analysis. There were four sessions with groups between 14 and 17 subjects. All the preliminary instructions and questions were projected on a large screen and the subjects kept a leaflet with the main instructions. Then each subject answered the questions on a computer. No oral communication was allowed between participants. The total time requested never exceeded one hour.

Each subject received an endowment of 10 € for her participation. She first answered hypothetical questions during 15 minutes in order to get trained with the procedure. Then she answered real questions and gave personal information. She was first confronted with 30 pricing tasks, then with 45 choosing tasks, all presented in random order. Subjects were never asked to set bidding prices and selling prices together in the same session. Two sessions each

were devoted either to bidding prices (30 subjects) or to selling prices (32 subjects). All prices were obtained with BDM mechanism. An example of the three decision tasks appears in figures 1, 2 and 3 (translated from French).

[Figures 1, 2 and 3 here]

Once a subject had answered all the questions, her decisions were printed and she left the lab. She drew one of her 75 decisions randomly and played for real money. When a choosing task was played, the outcome was determined by the draw of a number between 1 and 20 since all probabilities of outcomes were multiples of 0.05. For a pricing task, she drew one number among 100 (BDM mechanism). However, given the large variability of gains made possible by our experiment, we adjusted the interval in which the offer price used by the BDM mechanism could be randomly selected so as to impose no artefactual upper limit on the prices. For bidding prices, the random price was always drawn between 0.1 and 10 €. For asking prices, however, it was drawn between 0.1 and  $X$  € ( $X = 10, 20, 30, 40$ ) since the latter might be well in excess of the expected gain when the maximum gain reaches high values<sup>6</sup>. The gains varied between 6 and 31 €. Subjects earned 14.56 € on average (12.33 in the buying group and 16.65 in the selling group).

[Table 4 here]

The 75 decisions are classified in 15 sets which all follow the same pattern. In each set, a single \$-bet can be compared in a choice with three types of P-bets: a nondegenerate lottery (labelled  $P$ ), a small sure gain  $m$ , and a large sure gain  $M$ . Both  $P$  and  $m$  have systematically lower expected values than \$, while  $P$  has a higher expected value than \$. These 15 sets of

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<sup>6</sup> In a pilot experiment, all random offer prices were drawn between 0.1 and 10 €. This upper limit was clearly binding for the asking prices of a few lotteries yielding gains higher than 10 €, but never for the bidding prices. In the asking price sessions, the results were not different from those reported here when the 10€ limit was not binding but they were slightly different when the 10 € limit was binding.

options are described in table 5. For example, in the first set,  $\$ = (10; 0.8)$  is confronted with  $P = (8; 0.95)$ ,  $m = 7$  and  $M = 9$ . The fifteen  $\$$ -bets were selected so as to provide a wide variability in the objective probability and size of the maximum gain. The latter ranges from 10 to 40 € and probabilities of gain for the  $\$$ -bet range from 0.10 to 0.80.

Each set of options generates two evaluations ( $P$  and  $\$$ ) and three choices ( $\$$  or  $P$ ), ( $\$$  or  $m$ ), and ( $\$$  or  $M$ ). The purpose of this experimental design is fourfold. First, it allows comparison of PR when lottery values are elicited by minimum selling prices and by maximum bidding prices. This provides a decisive test of contingent weighting theory and regret theory. Second, it allows comparison of PR with PR1, which yields another decisive test of regret theory. Third, by allowing win probabilities to exceed one-half, we can test CC theory's prediction that PR can also be observed with rather high probabilities of winning with  $\$$ . Fourth, this specific design offers a very simple way of characterizing the EU-preference and thus testing CC theory, since the latter takes EU as the prior preference and makes novel predictions of PR conditional on this preference.

In this experiment, we use the BDM mechanism (after thoroughly explaining it to the participants) to elicit the true asking and bidding prices, and the random lottery incentive system to encourage subjects to answer all problems carefully as if they played each time a single gamble. These widely used procedures are incentive-compatible if subjects have EU preferences but their experimental validity is no longer guaranteed if subjects have non EU preferences. Charles Holt (1986), Edi Karni and Zvi Safra (1987) and Uzi Segal (1988) have shown, for instance, that these would be biased elicitation procedures if subjects had normative preferences which violate the independence and/or reduction axioms of EU theory and might lead to a spurious appearance of standard PR. As a result, Tversky et al. (1990) and Robin Cubitt et al. (2004) had to demonstrate the reality of PR by using another incentive mechanism which is immune from these criticisms, the ordinal payoff scheme. However, the

latter procedure can only elicit ordinal comparisons of the asking and bidding prices, and thus does not allow quantitative studies of each price and WTA/WTP disparity. Fortunately, we demonstrate the following proposition (see proof in appendix):

### **PROPOSITION 6**

*The BDM mechanism and the random lottery incentive system are incentive-compatible if subjects have preferences which satisfy CC theory.*

Thanks to this result, evidence of non rejection of CC theory on our experimental data should be conclusive. Moreover, if the theoretical predictions can be accepted, it will be possible to say that the three anomalies that we observe in a single task context, i.e. the PR phenomenon, WTA/WTP disparity and the endowment effect on simple bets, are not artefacts.

## **IV. Results**

### *A. Preference reversal and the WTA/WTP disparity*

The main results are summarized in tables 5 and 6. Frequencies of choice and average buying and selling prices of lotteries are indicated in table 5, whereas PR and PR1 rates conditional on choice appear in table 6 for the selling group and for the buying group.

[Tables 5 and 6 here]

As a check of reliability of these experimental data, none of the frequencies of the 45 choices that the selling group and the buying group had to make in separate sessions, shown in the rows of table 5, was found to be significantly different between groups at the 5 percent level. Such stability of choices across treatments is remarkable because the choice tasks were

executed *after* evaluations: thus the adoption of different ways of setting prices did not induce different ways of choosing.

[Figures 4 and 5 here]

However, the asking price is always significantly higher than the buying price in table 5. The ratio between WTA and WTP is 2.36 for \$ and 1.69 for *P*. Average asking and buying prices are correlated at 68 percent because they relate to the same lotteries, but the asking price is more than twice the buying price. This is a clear instance of the WTA/WTP disparity. Furthermore, asking prices are considerably closer than bidding prices to the expected value of the *P*-bet close to certainty. On the fifteen *P*-bets, the ratio of price to expected gain averages to 0.925 for asking prices and 0.525 for buying prices. These findings support CC theory's prediction of a discontinuity of the relation of buying prices with probabilities of winning close to certainty and continuity of asking prices. It should be noted that Casey (1994) had already observed that buying prices were far below expected values even for bets with probabilities of winning near one.

## **RESULT 1**

*WTA is higher than WTP in single valuation tasks, over and above what income effects alone can explain. The WTA/WTP disparity persists across variations in the probability of winning even when the latter gets close to one. There is a clear discontinuity in the relation of WTP with the probability of winning close to certainty, which contrasts with the continuity of WTA.*

We observe the standard PR in the selling group: 78 percent of the subjects who chose *P* gave a higher value for the asking price of \$. By contrast, only 11 percent of those who chose \$ reversed their preference in the pricing task. This well-known asymmetry of PR is predicted by CC theory under the joint assumption that EU-preference is approximated by expected

value for small and moderate gains. Since  $P$  has a systematically lower expected value than  $\$$  in our experiment, it can thus be assumed that most subjects must EU-prefer  $\$$ . Then, by looking at CC theory's predictions of PR in table 2a, we see that subjects EU-preferring  $\$$  and eventually choosing  $P$  should always indicate a higher asking price for  $\$$ , whereas subjects EU-preferring  $\$$  and choosing  $\$$  should never reverse their preference in revealing their minimum selling price. These two predictions, which form the essence of standard PR, are well corroborated in table 6.

Also notice that PR is not dependent on the win probabilities of lotteries, since the rate of PR in all cases is almost equal to 60 percent. For example, although  $\$$  has a win probability as high as 0.80 in set 1, the PR rate is 73 percent on this set. Conversely, sets 5, 6, 8 and 9 have lotteries  $P$  and  $\$$  with common structures but the rates of PR (79, 82, 60 and 82 percents) are not greater than the average rate, 82 percent.

## **RESULT 2**

*PR and PR1 can be observed with equal strength for large and for small win probabilities with  $\$, provided P remains safer than  $\$$  and both have close expected values.$*

An additional prediction reported in table 3a is that the standard PR extends to a single lottery in the selling price condition. Thus PR rates can be compared with PR1 rates with the  $(m, \$)$  pair. As expected, table 6 demonstrates that these rates of PR1 with a single lottery mimic the rates of PR with two lotteries. A further prediction of CC theory summarized in table 3a is that the standard PR does not hold any more when the sure gain (or the P-bet) is EU-preferred to the  $\$$ -bet, whether it is actually chosen or not. Since  $M$  was chosen to be larger than the expected value of  $\$, it can be assumed that most subjects EU-prefer  $M$  to  $\$. It is possible to verify in table 6 that the asymmetry of PR1 which prevailed with pairs  $(m, \$)$  is$$

no longer observed with pairs  $(M, \$)$ . In the latter case, the rate of PR1 is just about 50 percent whether individuals opted for  $M$  or for  $\$$ . All these results are unaffected by large variations in the probability of winning with  $\$$ , ranging from 0.1 to 0.8. This is in line with CC theory's predictions, but contrasts with previous experimental work on PR where the win probability for  $\$$  never exceeded one-half.

In the buying group, patterns of PR and PR1 are totally different. First, PR is predicted under risk neutrality for the  $\$P$  decision path and not predicted in the  $\$\$$  decision path. However, risk aversion might partially offset these predictions and make the theoretical predictions ambiguous. The rows labeled " $P$ " in table 7 show that a higher proportion of deviators (57 percent of the  $\$P$ -subjects) than non-deviators (36 percent of the  $\$\$$ -subjects) exhibit PR, but this difference is not very large. Predicted PR1 is much more clear-cut because the discontinuity of buying prices around certainty introduces a downward bias in the price of  $\$$  relative to  $P$  of a higher order of magnitude than potential effects of income and risk aversion. Consequently, a large majority of  $\$$ -choosers indicated a lower bidding price for  $\$$  than the sure value of  $P$  whereas few  $P$ -choosers reversed their preference in the bidding task. These conclusions are little affected by the size of the sure gain, and hold in a like manner in the rows of table 7 labeled " $m$ " and " $M$ ".

### **RESULT 3**

*In the minimum selling price condition, PR1 replicates standard PR with the same frequency.*

*In the maximum buying price condition, PR1 differs markedly from PR. PR1 then essentially concerns subjects choosing the risky bet.*

#### **B. Pricing and the endowment effect**

Since all bets imply small or moderate gains – ranging from 2 to 40 € –, it is reasonable to linearize the utility of wealth and omit income effects as the main explanation for the WTA/WTP disparity. The ratios of average asking and bidding prices to each bet's expected value are plotted on Figure 4 against the probability of winning. If income effects are omitted, equation (10) predicts that the asking price ratio follows a declining hyperbola that converges to one when the win probability approaches one and that the bidding price ratio remains constant below one, and equal to the prior's weight. Both predictions are strikingly confirmed by figure 4, and the constancy of the buying price ratio further suggests that the prior's weight is independent of the probability of winning. Thus figure 4 offers a nice illustration of the discontinuity of the relation of buying prices to win probabilities close to certainty and the continuity of asking prices. By the same token, CC theory's explanation of the endowment effect for seemingly riskless goods by the underestimation of bidding prices receives excellent support whereas the loss aversion explanation by the overestimation of asking prices is rejected by the data. Clearly, our subjects did not systematically overestimate the value of an almost sure beneficial bet when they owned it but they underestimated the value of this bet when they did not own it.

However, the omission of risk aversion and income effects can be misleading. If the latter assumption were true, equation (11) would allow us to compute the objection's weight  $1 - \mu$  as the ratio of the WTA/WTP disparity to the maximum gain. Figure 5 plots the latter ratio derived from each of the 30  $P$  and  $\$$  bets used in the experiments as a function of the maximum gain. The curve has a smooth convex shape on the [2, 40] interval (in €), but the ratio doesn't keep a constant value: it stays around 0.2 in most of the interval but rises to much higher values when gains fall below 10 €. Accounting for risk aversion and income effects helps to reconcile these observations with the assumed constancy of the objection's weight (at least over a class of related decision problems).

A higher-order approximation for asking and bidding prices can be derived from a second-order Taylor expansion of equations (6) and (8), when experimental gains are small relative to subjects' endowed wealth. Let  $\bar{p}_a$  ( $\bar{p}_a = (1 - \mu(1 - r))y$ ) and  $\bar{p}_b$  ( $\bar{p}_b = \mu r y$ ) respectively designate the asking price and the bidding price for risk-neutral subjects, then develop (6) around  $w + \bar{p}_a$  and (8) around  $w - p_b + \bar{p}_b$  at the second order. After simplification, we generalize the Arrow-Pratt's approximation

$$(12) \quad \Pi_a \equiv \bar{p}_a - p_a = -\frac{1}{2} \frac{U''(w + \bar{p}_a)}{U'(w + \bar{p}_a)} \text{Var}_a^*(B)$$

$$\text{with } \text{Var}_a^*(B) = (1 - \mu(1 - r))(y - \bar{p}_a)^2 + \mu(1 - r)\bar{p}_a^2 \\ = \mu(1 - r)(1 - \mu(1 - r))y$$

$$\text{and } \Pi_b \equiv \bar{p}_b - p_b = -\frac{1}{2} \frac{U''(w - p_b + \bar{p}_b)}{U'(w - p_b + \bar{p}_b)} \text{Var}_b^*(B)$$

$$\text{with } \text{Var}_b^*(B) = \mu r (y - \bar{p}_b)^2 + (1 - \mu r)\bar{p}_b^2 \\ = \mu r (1 - \mu r) y^2$$

$\text{Var}_a^*(B)$  and  $\text{Var}_b^*(B)$  are the *perceived variances* of (the Bernoulli) distribution  $B$  in which the prior probabilities of the relevant objection state (which differ for tasks  $a$  and  $b$ ) have been revised according to (1). Since  $\bar{p}_a$  and  $\bar{p}_b - p_b$  are both small in front of  $w$ , the two Arrow-Pratt coefficients of absolute risk aversion reported in (12) only deviate from  $-\left(\frac{U''(w)}{U'(w)}\right) \equiv \alpha$  by a first-order term. Hence, we derive from (12) a first-order approximation of the *ratio* of the WTA/WTP disparity with the maximum gain

$$(13) \quad \frac{p_a - p_b}{y} = 1 - \mu + \alpha \mu (1 - \mu) \left(r - \frac{1}{2}\right) y$$

This formulation applies to most experimental conditions where the experimental gains are small relative to subjects' initial wealth. Moreover, it does not rely on the whole utility of wealth curve but only on a fixed risk aversion parameter  $\alpha$ . Related approximations can be

derived from (12) for the asking price and for the buying price. All these price equations are non-linear in the two parameters  $\alpha$  and  $\mu$ .

As a first step, we estimate a linear equation by equating the asking and bidding prices and the WTA/WTP disparity with their risk-neutral values. We write concisely

$$\begin{aligned} \text{Price or WTA - WTP disparity} &= \beta_0 + \beta_1 r + \beta_2 y + \beta_3 r y \\ \text{with } \begin{cases} \beta_0 = \beta_1 = 0 \\ \beta_2(p_b) = 0 \\ \beta_2(p_a) = \beta_2(p_a - p_b) = 1 - \mu \\ \beta_3(p_a - p_b) = 0 \end{cases} \end{aligned}$$

The result of these three linear estimations is shown in table 8. Only one of the three constant terms is significant at the 5 percent level, which is not very far from the prediction. In further conformity with theoretical predictions, the three coefficients of  $r$  and the coefficient of  $y$  in the bidding price equation are insignificant. Moreover, the coefficients of  $y$  are not significantly different, as they should be, in the asking price and WTA/WTP disparity equations. Their common value yields for the objection's weight an estimate of 0.15. The coefficients of the interaction terms in  $ry$  are all positive and significant at the 1 percent level. This is in line with predictions for both price equations. However, the same coefficient should be null for the WTA/WTP disparity equation under the assumption of risk neutrality. Therefore, this simplifying assumption is rejected by the data and risk aversion is validated.

Indeed, equation (13) demonstrates that the WTA/WTP disparity ratio should rise, under risk aversion, when the win probability approaches one, a situation of particular relevance for  $P$ -bets. Therefore, we ran the following simple linear regression on the sample composed of the 30  $P$  and  $\$$ -bets

$$\frac{P_a - P_b}{y} = \beta'_0 + \beta'_1 \left(r - \frac{1}{2}\right) y$$

Equation (13) was used to identify the two parameters  $1 - \mu = \beta'_0$ ,  $\alpha = \beta' / (\beta'_0(1 - \beta'_0))$ .

The regression coefficients are both significant at the 1 percent level with an  $R^2$  of 0.2525. From these values, we compute  $1 - \mu = 0.307$  ( $\mu = 0.693$ ),  $\alpha = 0.0573$ . A nonlinear regression yielded exactly the same values for these parameters. Thus, the objection's weight estimate is doubled when risk aversion is taken into account. With an average expected gain of all bets around 12€, the partial risk aversion parameter estimate of  $12\alpha \cong 0.7$  is quite reasonable. These estimates of risk aversion and the objection's weight do not rest on the parametric form adopted for the utility function.

## V. Conclusion

We presented a new theory of choice under risk, cognitive consistency (CC) theory (Lévy-Garboua 1999), which can predict both standard and non-standard cases of PR, the WTA/WTP disparity and the endowment effect in the context of single tasks. CC is a theory of the decision process based on the sequential perception of two dissonant cognitions: the prior preference (EU in the context of single choices) and objection to the latter. Pairwise choices and values have been described by an objection-dependent expected utility (ODEU) which is an average of two EUs, prior EU and the objection's EU, with an individual-specific weight assumed constant across lotteries. Therefore, CC generalizes EU by letting the "objection's weight" be positive. Under CC theory, the disparity between minimum selling prices and maximum buying prices of bets merely results from the common use of descending auctions for the valuation of goods owned and ascending auctions for the valuation of goods not owned. The same assumption is sufficient to predict both standard and non standard cases of PR, that is, PR with asking prices and with bidding prices and PR1 with a single lottery confronted to a sure outcome. Differences in PR patterns with asking prices and with bidding prices are not well explained by contingent weighting theory, and PR1 with a single lottery

plainly contradicts the regret theory explanation of PR. Moreover, under CC theory, the endowment effect arises as a surprising consequence of the WTA/WTP disparity when the probabilities of winning get close to certainty. Whereas the asking price converges toward the gain, the bidding price of an almost sure gain stays at a lower value if the objection's weight is positive. The endowment effect is created, not by the overestimation of the good's value by owners (which is the loss aversion explanation), but by the underestimation of the good's value by non-owners.

A new experiment was designed to test the predictions of CC and alternative theories of PR and the endowment effect. The experimental results strongly support CC theory's predictions and reject the regret theory explanation for PR and the loss aversion explanation for the endowment effect. Finally, we estimated the average objection's weight (constant across lotteries in our experiments), about 0.3 and partial risk aversion about 0.7 without making assumptions on the parametric form of the utility function.

All propositions and results found in this paper have been established in the context of single tasks and may not survive to the repetition of tasks, that is, precisely the context in which John List (2003) and Plott and Kathryn Zeiler (2005) found substantial erosion of the endowment effect and of the WTA/WTP disparity. The effect of experience acquired in the market or in the experiment itself is consistent a priori with our analysis of the endowment effect since experience reduces the trader's uncertainty about the good's value. Although CC theory is a Bayesian theory of sequential perception which could accommodate learning (see Lévy-Garboua 1999), we did not examine the effect of experience in this specific study because the anomalous behavior of inexperienced subjects follows systematic patterns which deserve explanation.

Parsimony is a remarkable feature of CC theory since the latter generalizes EU with a single parameter added. This parameter explains many anomalies or paradoxes: PR, the

WTA/WTP disparity and the endowment effect which were the objects of our present study, but also the Allais paradox (Blondel, 2002; Lévy-Garboua, 1999), cognitive dissonance, the reflection effect (Lévy-Garboua and Blondel, 2002) among others. Indeed, CC theory had already been validated by a test on pairwise choices between EU and five non-EU theories (Blondel, 2002). To our knowledge, only the “extended prospect theory” in the Schmidt et al. (2005) version can also explain some of these phenomena, at the price of adding at least two parameters to EU, one for the probability weighting function, and one for loss aversion. That only one individual parameter in addition to risk aversion can explain so many puzzles underlines that the failures of EU could have, after all, a single origin.

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## TABLES

TABLE 1. CHOOSING BETWEEN A P-BET AND A \$-BET IN AN ACTION FRAME

	$r_S$	$r_P - r_S$	$1 - r_P$
$P$	$y_P$	$y_P$	0
$\$$	$y_S$	0	0

TABLE 2A. PR WITH TWO BETS: ASKING PRICES

		<b>ODEU</b>	
		<b>P</b>	<b>\$</b>
<b>EU</b>	<b>P</b>	Impossible	Impossible
	<b>\$</b>	<b>Always true</b>	Impossible

TABLE 2B. PR WITH TWO BETS: BIDDING PRICES

		<b>ODEU</b>	
		<b>P</b>	<b>\$</b>
<b>EU</b>	<b>P</b>	Impossible under wealth neutrality	<b>Always true under wealth</b> <b>neutrality</b>
	<b>\$</b>	<b>Always true under wealth</b> <b>neutrality</b>	Impossible under wealth neutrality

TABLE 3A. PR1 WITH A SINGLE BET: ASKING PRICES

		<b>ODEU</b>	
		<b>P</b>	<b>\$</b>
<b>EU</b>	<b>P</b>	Impossible	Impossible
	<b>\$</b>	<b>Always true</b>	Impossible

TABLE 3B. PR1 WITH A SINGLE BET: BIDDING PRICES

		<b>ODEU</b>	
		<b>P</b>	<b>\$</b>
<b>EU</b>	<b>P</b>	Impossible <sup>a</sup> without restriction	<b>Always true under risk- neutrality</b>
	<b>\$</b>	Impossible under risk- neutrality	<b>Likely under experimental conditions</b>

<sup>a</sup> All conclusions only hold if individuals are unsure of their EU-preference.

TABLE 4. LOTTERIES USED IN THE EXPERIMENT (WITH EXPECTED VALUES)

Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
m	7	6	5	4	3.5	2.5	1.8	7	5	3.5	1.8	5	2.5	7	3.5
M	9	8	7	6	4.5	3.5	2.2	9	7	4.5	2.2	7	3.5	9	4.5
P	8;0.95	7;0.95	6;0.95	5;0.95	4;0.95	3;0.95	2;0.95	8;0.9	6;0.9	4;0.9	2;0.9	7;0.8	3;0.9	9;0.8	4,5;0.8
EV(P)	7.6	6.65	5.7	4.75	3.8	2.85	1.9	7.2	5.4	3.6	1.8	5.6	2.4	7.2	3.6
\$	10;0.8	10;0.7	10;0.6	10;0.5	10;0.4	10;0.3	10;0.2	20;0.4	20;0.3	20;0.2	20;0.1	30;0.2	30;0.1	40;0.2	40;0.1
EV(\$)	8	7	6	5	4	3	2	8	6	4	2	6	3	8	4

TABLE 5. FREQUENCIES OF CHOICE AND AVERAGE PRICES OF BETS <sup>a</sup>

	Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	All
<u>Selling group (N=32)</u>																	
P > \$ (percent)		47	53	50	41	44	34	41	47	53	47	38	53	47	41	59	46
m > \$ (percent)		37	37	38	34	41	34	19	38	53	34	31	47	41	53	47	39
M > \$ (percent)		66	62	72	75	69	47	31	59	69	63	38	81	56	78	66	62
Average \$ price (€)		7.1	6.6	6.0	5.5	4.6	4.0	3.5	8.3	6.8	6.0	4.7	8.2	7.3	11.2	9.6	6.6
Average P price (€)		6.6	5.9	5.1	4.4	3.5	2.7	2.0	6.4	4.7	3.2	1.9	5.4	2.7	6.4	3.6	4.4
<u>Buying group (N=30)</u>																	
P > \$ (percent)		40	47	57	37	57	50	37	50	37	30	47	57	63	40	60	47
m > \$ (percent)		40	33	30	30	37	30	33	50	53	30	33	50	47	53	47	40
M > \$ (percent)		87	77	83	77	67	67	33	67	77	73	40	73	60	77	67	68
Average \$ price (€)		4.1	3.2	2.9	3.0	2.6	2.2	1.5	3.5	2.9	2.6	1.6	3.1	2.1	3.6	2.5	2.8
Average P price (€)		4.5	3.9	3.4	2.7	2.1	1.4	0.9	4.1	3.4	1.9	0.7	2.9	1.3	3.8	1.6	2.6

<sup>a</sup> All of the percentages are not significantly different (at 5 percent) between the two groups. All of the buying prices are significantly lower (at 5 percent) than the selling prices.

TABLE 6. PERCENTAGES OF PR AND PR1 <sup>a</sup>

	Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	All
<u>Selling group (N=32)</u>																	
PR / P > \$		73	82	69	85	79	82	69	60	82	87	83	65	80	92	89	78
PR1 / m > \$		67	67	67	91	77	91	83	<u>33</u>	76	91	90	80	100	<u>71</u>	87	78
PR1 / M > \$		10	30	48	38	55	<u>67</u>	<u>60</u>	<u>42</u>	36	55	83	<u>50</u>	83	52	52	51
PR / \$ > P		24	13	19	11	17	5	0	18	0	6	5	13	0	21	8	11
PR1 / \$ > m		25	20	10	0	26	5	4	10	20	5	9	24	11	13	12	13
PR1 / \$ > M		100	92	89	75	50	35	27	54	40	33	40	33	14	14	9	47
<u>Buying group (N=30)</u>																	
PR / P > \$		42	43	53	45	59	73	100	40	36	56	71	53	79	45	67	57
PR1 / m > \$		0	0	11	22	9	33	20	0	13	44	20	13	36	6	21	17
PR1 / M > \$		0	0	0	4	10	20	10	5	0	18	25	9	6	4	10	8
PR / \$ > P		44	56	62	32	31	13	11	60	47	33	13	46	18	37	33	36

PR1 / \$ > m	94	90	90	57	63	57	60	93	79	62	60	73	69	86	75	74
PR1 / \$ > M	100	100	100	100	80	90	80	100	100	100	83	100	92	83	90	93

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<sup>a</sup> In the selling group, all percentages (in percent) are not significantly lower (at 5percent) for PR1 / m > \$ than for PR / P > \$ , except for the underlined ones. However, all percentages are significantly lower (at 5 percent) for PR1 / M > \$ than for PR / P > \$ , except for the underlined ones.

TABLE 7. LINEAR ESTIMATION OF WTA, WTP AND WTA/WTP DISPARITY (IN BOLD THE PREDICTED VARIABLES)

Dependent variable	Causal variables			
	Constant	$r$	$y$	$r.y$
$P_a$	0.62*	0.08	<b>0.15**</b>	<b>0.60**</b>
$P_b$	0.68	-0.38	-0.01	<b>0.48**</b>
$P_a - P_b$	-0.07	0.46	<b>0.16**</b>	<b>0.13**</b>

\* Significant at 5 percent

\*\* Significant at 1 percent.

## FIGURES

FIGURE 1. CHOICE BETWEEN TWO OPTIONS

**CARNOT**

You have to choose between two tickets and you will draw a number from 1 to 20.

→  = 5 euros with numbers 1 à 19

→  = 10 euros with numbers 1 to 10

Choose your ticket.




FIGURE 2. MINIMUM SELLING PRICE

**NASH**

- You must indicate your **minimum selling price** for a ticket of lottery offering 10 euros with the numbers 1 to 10 amongst 20 :  
→ Your price =  euros.
- The buying price, included between 0.1 € and 10 €, will be equal to a number drawn from numbers 1 to 100, multiplied by 10 cent of €.  
→ Your price > buying price : you do not sell  
→ Your price ≤ buying price : you sell at the buying price.

Note : it is your interest to indicate your real price and not a "strategic" price because you may be prevented from selling at an interesting price (if your price is too high) or you may be forced to sell at a too cheap price (if your price is too low).




FIGURE 3. MAXIMUM BUYING PRICE

**NASH**

- You must indicate your **maximum buying price** for a ticket of lottery offering 10 euros with the numbers 1 to 10 amongst 20 :  
 → Your price =  euros.
- The selling price, included between 0.1 € and 10 €, will be equal to a number drawn from numbers 1 to 100, multiplied by 10 cent of €.  
 → Your price  $\geq$  selling price : you buy at the selling price  
 → Your price  $<$  selling price : you do not buy.

Note : it is your interest to indicate your real price and not a "strategic" price because you may be prevented from buying at an interesting price (if your price is too low) or you may be forced to buy at a too expensive price (if your price is too high).

▶

FIGURE 4. RATIOS OF AVERAGE ASKING ( $\blacktriangle$ ) AND BIDDING PRICES ( $\bullet$ ) TO EXPECTED VALUE AS A FUNCTION OF THE PROBABILITY OF WINNING

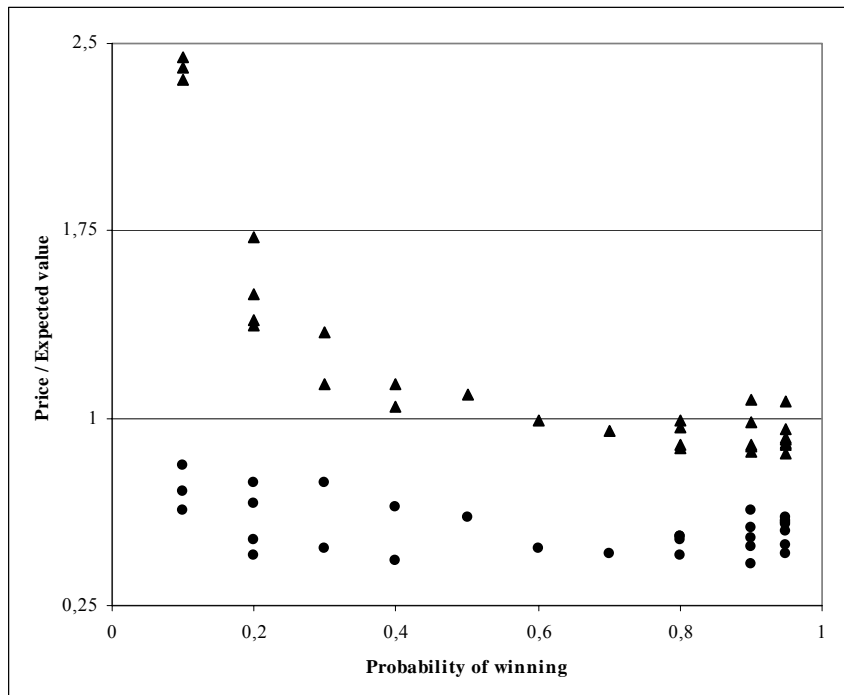
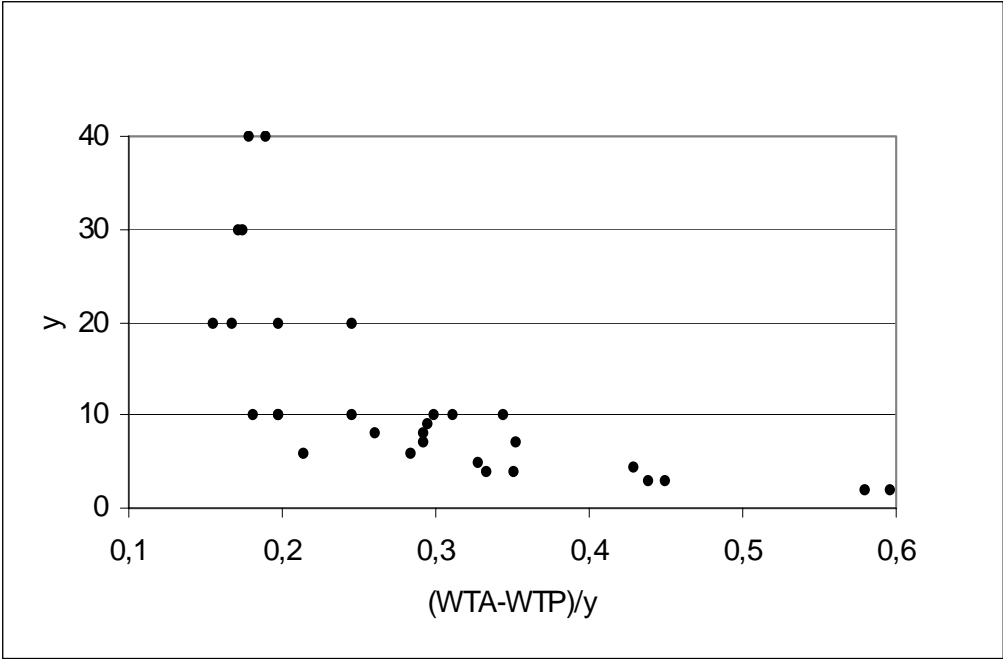


FIGURE 5. RATIO OF AVERAGE WTA/WTP DISPARITY TO GAIN AS A FUNCTION OF GAIN



APPENDIX A: PROOF OF PROPOSITION 2

Proof of proposition 2a:

(i) Assume first:  $EU(\$) > EU(P)$ . This entails:  $\mu EU(\$) + 1 - \mu > \mu EU(P) + 1 - \mu$   
 $> \mu EU(P) + (1 - \mu)x$ .

Hence, by (6),  $p_a(\$) > p_a(P)$ . PR holds in this case iff  $P \succ \$$ , that is,  $ODEU(P/\$) > ODEU(\$/\$)$ , which requires  $\mu < 1$ .

(ii) Assume then:  $EU(\$) < EU(P)$ .  $\$ \succ P$  iff  $ODEU(\$/P) > ODEU(P/P)$ , that is,  $\mu EU(\$) + 1 - \mu > \mu EU(P) + (1 - \mu)x$ , or by (6),  $p_a(\$) > p_a(P)$ . PR can never be observed in this case.

Table 2a summarizes the conclusions drawn from (i) and (ii).

Proof of proposition 2b:

If  $p_b(\$) > p_b(P)$ ,  $U(w - p_b(\$)) < U(w - p_b(P))$  and  $EU(w - p_b(\$) + \$) > EU(w - p_b(P) + P)$  with (8). Hence, we get  $EU(w - p_b(P) + \$) > EU(w - p_b(P) + P)$ . The last condition means that  $\$$  is EU-preferred to  $P$  when wealth equals  $w - p_b(P)$ . If EU-preference is wealth-neutral, we can say that  $\$$  is EU-preferred to  $P$  at all levels of wealth and this property carries over to ODEU-preference. Consequently, PR is obtained if  $P \succ \$/\$$ ; but there is no PR if  $\$ \succ P/\$$ .

The whole argument is symmetric in  $P$  and  $\$$ . This shows the proposition, which is summarized by table 2b.

APPENDIX B: PROOF OF PROPOSITION 3

Proof of proposition 3(a) in the selling price condition is immediate and given in the text. We give here detailed proofs of proposition 3(b) in the buying price condition. The four decision paths **PP**, **\$P**, **P\$**, **\$\$** will be examined successively (EU-preference-the prior- is indicated by

the first symbol and revealed preference by the second). We always assume here that  $P$  is a sure gain and  $\mu < 1$ .

(i) **PP**: We show that **PP**  $\Rightarrow$  (no PR1). Since  $ODEU(P) \equiv U(w + y_p)$  if  $P$  is a sure outcome, the two preference conditions implied by **PP** that derive from (4) and EU-preference are the following:  $U(w + y_p) \geq \mu EU(\$) + (1 - \mu)U(y_s)$  and  $U(w + y_p) > EU(\$)$ . The latter inequality can be skipped because it is implied by the first. By using (6), we express the right-hand member of the first condition as  $U(w + p_a(\$))$ , and simply get  $y_p \geq p_a(\$)$ . Since  $p_b(P) \equiv y_2$  when  $P$  is a sure outcome, the latter inequality means that  $p_b(P) \geq p_a(\$)$ , which rules out PR1 in this case.

(ii) **\$P**: Under risk-neutrality, **\$P**  $\Rightarrow$  (no PR1). We derive from (3) and EU-preference that  $P$  is revealed preferred to  $\$$  by the **\$P** decision path iff  $\mu r_s < x < r_s$ . PR1 would imply  $p_b(P) < p_b(\$)$ , that is,  $y_p < p_b(\$)$  since  $P$  is a sure outcome. Dividing both members by  $y_1$  and using (10), we would get if  $U'' = 0$ :  $y_p/y_s = x < \mu r_s$  which contradicts ODEU-preference for  $P$ .

(iii) **P\$**: Under risk neutrality, **P\$**  $\Rightarrow$  PR1. We derive from (4) and EU-preference that  $\$$  is revealed preferred to  $P$  by the **P\$** decision path iff  $\mu r_s + 1 - \mu > x > r_s$ . We then show that (no PR1), *i.e.*  $p_b(\$) > y_p$ , leads to a contradiction under risk neutrality. Indeed, by dividing both members of the latter inequality by  $y_s$  and using (10), we would get if  $U'' = 0$ :  $y_p/y_s = x < \mu r_s$ , which contradicts  $x > r_s$ .

(iv) **\$\$**: If  $P$  and  $\$$  yield close expected gains, PR1 is likely to occur under risk neutrality in following the **\$\$** decision path when  $\mu$  is small enough. The preference conditions derived from (3) and EU-preference are in this case:  $r_s > x$  and  $\mu r_s + 1 - \mu > x$ . We can skip the second condition that is implied by the first. We then show that (no PR1), *i.e.*  $p_b(\$) > y_p$ , is

unlikely under risk neutrality. Indeed, by dividing both members of the latter inequality by  $y_1$  and using (10), we would get if  $U'' = 0$ :  $y_P/y_S = x < \mu r_S$ . In the experimental conditions,  $x$  is close to  $r_1$  since  $P$  and  $S$  yield close expected gains. Thus, if  $\mu$  is small enough, it is unlikely that  $x < \mu r_S$  even though  $x < r_S$ , and PR1 is likely to occur.

These conclusions are summarized in table 3b.

#### APPENDIX C: PROOF OF PROPOSITION 5

With the random lottery incentive system, subjects answer  $N$  problems one of which will be selected randomly at the end of the session and played for real money. All problems can be framed as choice problems, including valuation problems. Letting the problem-specific alternatives be  $\{A_{ij}, i = (1, \dots, N), j = (1, \dots, m(i))\}$ , EU theory would predict the choice of the  $N$  problem-specific alternatives  $A_{ij^*(i)}$  that maximize  $\frac{1}{N} \sum_{i=1}^N EU(A_{ij})$ . Thanks to the additive separability, the best answer consists of finding, step by step, the  $N$  maxima for  $EU(A_{ij})$ , which is incentive compatible under EU theory. In particular, choice and valuation questions about the same lottery become separable (Holt, 1986). Under CC theory, an individual takes these EU preferences as priors and then looks for objections. Since each prior is the problem-specific EU-maximizing lottery, the related objection is the one which arises in a single choice, and the  $N$  preferred lotteries revealed by the random lottery incentive system are still incentive compatible under CC theory.

An additional incentive system, the BDM mechanism, applies to valuation problems. Before reporting their minimum selling price, for instance, subjects are told that they would have to sell the lottery if and only if the selling price which they had previously reported was lower than the “offer price”,  $p$ . In the BDM mechanism, the offer price is generated by a random device, and this is common knowledge. Let the offer price be a positive random

variable on the interval  $(0, P)$  with distribution  $dF(p)$ . With the BDM mechanism, individuals must choose an asking price  $p_a$  at which they are uncertain to sell. They face a set of lotteries indexed by the reported value  $v$ , which we denote  $(B, F(v); |p, dF(p)|_v^P)$  since they will have to play  $B$  (with probability  $F(v)$ ) if the random offer price falls below the reported value and receive the random offer price  $p$  if the latter takes any value at least as high as the reported value (Karni and Safra 1987, John Horowitz 2005). The “minimum selling price” framing suggests that  $p_a$  be determined by starting from a high value, say  $P$ , and lowering  $v$  to get increasingly-preferred lotteries until the preference order is reversed. The reported value for which PR is obtained determines the asking price. That is,

$(B, F(v); |p, dF(p)|_v^P) \succeq (B, F(v + dv); |p, dF(p)|_{v+dv}^P)$  holds for all  $v \geq p_a$  if  $dv > 0$ , and for all  $v < p_a$  if  $dv < 0$ . Under CC theory, the sure thing principle applies to choices between risky actions with well-specified states of the world. Both the prior EU preference and the objection are unaffected by the cancellation of states of the world in which alternative actions share common outcomes. By canceling common outcomes  $(B, F(v))$  and  $|p, dF(p)|_v^P$  and letting  $dv \rightarrow 0$ , we derive from the preference relation listed above:  $v \succeq B$  iff  $v \geq p_a$ . Therefore, the asking price revealed by the BDM mechanism is incentive compatible under CC theory, as under EU theory.