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## DIRECTIONAL DISTANCE FUNCTION FOR FDH TECHNOLOGIES WITH AN APPLICATION TO TUNISIAN PUBLIC HOSPITALS

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## **Abstract:**

Tunisian's public hospitals have been often deemed inefficiency, yet unfortunately no empirical study has demonstrated the breadth and/or depth of this inefficiency. To address this issue, we develop a non parametric methodology to measure efficiency at patient level for five cardiology wards. This methodology is based on the estimation of the input directional distance function, using a recent developed linear form of free disposal hull model. Results show that, on average, each cardiology ward could save more than 50% of each input; whereas at the patient level, illness severity and patients' profiles were associated with inefficiency. At the ward level, practice patterns and wards missions explain disparities on inefficiency. By defining sub-specializations or standard procedures for cardiology wards, we demonstrate where the public hospitals can save resources when treating these patients.

**Keywords:** Directional Distance Function, Free Disposal Hull, Tunisian's public hospitals

## **1. Introduction**

Tunisia's health system is a mixture of public hospitals and for-profit private clinics. The network of public hospitals provides 89% of total inpatient days, and employs 98% of paramedical (nonmedical) staff, 55 % of physicians, 30 % of dentists and 18% of pharmacists. It is structured into three levels of health care provision: the primary level (2068 health centers and 118 local hospitals and peripheral maternity) offering preventive and primary care. The secondary level (32 regional hospitals) provides specialized care, with technical equipment and sufficient beds. The tertiary level (22 university hospitals) provides highly specialized care and emergency care to patients and to those patients transferred from other facilities, as well as providing teaching for medical and paramedical students and medical research.

Health professionals are full-time salaried employees in the hospital. However, in some circumstances, university physicians (professors) are authorized to treat private patients through one day's consultations in the public hospital and one day on the private facilities for surgical or medical procedures. The public hospital is financed subject to an annual fixed budget (63%), and participation of national funds of social insurance (22%) and returns coming from patients' direct payments. The hospital budget sets forth estimated expenditures and revenues for the coming year. This budget is to conform to public accounting principles and has two sections: i) the operating section deals with current activity, including the day-to-day running of the hospital and financial management; ii) the investment section deals with operations leading to an increase in durable capital assets requiring depreciation. The budget and recruitment are fixed at the Ministry of public health level.

University hospitals contain 51% of total public hospital beds (8305 beds) with a population ratio equal to 0.87 beds per 1000 inhabitants (Banque Mondiale, 2006). These hospitals are corporate bodies governed by public law and are responsible for providing a public service. They are however, subject to various forms of public accounting, supervision and financial control. There are 177 wards at these hospitals, of which 40 are medical speciality and employ 70% of public specialized physicians. They offer highly specialized health care requiring sophisticated equipment; and they absorb 50% of the total operating budget devoted to all public hospitals; the health professional costs are estimated at 54,1% of total expenditures of public hospitals for the year 2002. For all hospitals, the total budget has experienced a

growth rate of 190%, which is related to the priority placed on funding issues according to the recent reform of hospitals.

The aim of reforms introduced over the last decade is to improve the financing mechanisms and hospital performance. At the tertiary care level, major organizational and management reforms were introduced in university hospitals. The July 1991 law has changed the legal status of teaching hospitals into autonomous entities - "établissements publics de santé" (EPS). This autonomy is not total for some aspects like human resources. Experience to date demonstrates a marked improvement in the EPSs' performance: activities have increased by 15-20%, particularly in the outpatient department with no significant increase in personnel, and the average length of stay has dropped from 8.5 to 7.5 days (Achouri H, 2001). Many management tools have been introduced like management procedures manual, accounting and financial management system and a computerized management information system. A new billing system for inpatient and outpatient services has been introduced using a simplified version of a Diagnostic Related Grouping system. This newly piloted payment mechanism and billing system have enabled the hospitals to increase their revenues.

A reform of the health insurance funds under the Social Security System was also conceived to enhance basic coverage combined with optional supplementary coverage, access to all providers and financing mechanisms. The tariffs applied in public hospitals were revised to bring them more in line with the real costs of services. The social security funds have entered into agreements with public hospitals which allow for lump-sum payments for the services provided to both outpatients and inpatients with social insurance coverage. This has also made it possible to increase the contributions of the social security funds to the hospitals' budgets.

The wards of these hospitals can be considered as autonomous entities represented by the ward chair who defines the production process according to medical staff objectives. The autonomy is observed on decisions regarding service operations and the global management of ward (human and financial resources, the use of equipment and installations). The basic question faced by the ward chairs is whether the production of health care is technically efficient and how much we can save for a given level of outputs production? At the ward level, the measurement of technical efficiency will be related to health care production process and it will be called clinical efficiency (Chilingerian J A, 2004). For the case of public wards, the input based efficiency is adopted here and justified for university hospital. We use

the measurement of input efficiency because of the higher level of occupancy rate of beds (average value of 80%) and activities of medical researches and training.

The focus of our study is to measure input efficiency of five cardiology wards located in the district of Tunis (capital city plus three nearby “governorates”). The novel prospective of this study is the development of non parametric model to measure ward efficiency at the patient level. We also investigate factors affecting the variation of wards inefficiencies. The reference technology is constructed from observed outputs and inputs using free disposal hull (FDH) model (Deprins et al, 1984). The FDH technology is based on a representation of the production technology without the convexity assumption applied in other non-parametric techniques and is appropriate for the efficiency measurement at the patient level. Efficiency, define for our purposes, is estimated relative to technology frontier using a non-radial measure as suggested by Färe and Lovell (1978). The directional distance function is used to estimate distance between inefficient production unit and its reference unit on the frontier technology Luenberger (1992). The directional distance function is a complete representation of a technology exhibiting free disposal of inputs and outputs (Chambers et al., 1998) and it's equal to technical efficiency.

The directional function is so general that it encompasses all known distance functions representations of the technology as special cases. We estimate the directional distance function using the linear FDH model (Agrell and Tind, 2001; Leleu, 2006). This approach has characteristics that are especially useful in measuring hospital efficiency. First, the non-parametric technology can easily deal with multiple outputs and inputs. Second, technical efficiency scores are measured in a non-radial manner at the patient level which can then be aggregated at the ward level via the use of the directional distance function. Further, based on Koopmans (1951) notion of technical efficiency, the non-radial input efficiency measure is appropriate because it can account for zero values of inputs which is required for this study.. If we had used the radial input efficiency measure as defined by Debreu (1951) and Farrell (1957), all production plans with at least one zero value inputs will be considered as efficient in the input directional distance function. Third, the use of control variable allows for the possibility that wards facing different diagnosis and different operating environments may employ different technologies.

The paper proceeds as follows. In the second section, we develop the model of the non-radial technical efficiency measure using the FDH linear programming techniques. In the third

section, we present the data used in the empirical efficiency measure. In the fourth section, we present the FDH results followed a discussion of the empirical application in section five. The last section summarizes our conclusions.

## 2. Linear Free Disposal Hull

We formulate the linear form of FDH model to estimate the technical efficiency using the input directional distance function as a complete input set. First, let us consider a production technology defined from a set of  $K$  observed production plans  $\{(x_k, y_k), k = 1, \dots, K\}$ . Second, producers use a nonnegative vector of input  $(x = (x_1, \dots, x_N) \in \mathfrak{R}_+^N)$  to produce a nonnegative vector of output  $(y = (y_1, \dots, y_M) \in \mathfrak{R}_+^M)$ . The production technology is given by its input set,  $L(y) = \{x \in \mathfrak{R}_+^N \mid x \in L(y)\}$ .  $L(y)$  describes the set of inputs vectors that are feasible for each output vector. Under the axiomatic approach to production theory, the input set is assumed to satisfy certain axioms, defined by Shephard (1953, 1970) and extended by Färe et al., (1994):

- L1.  $\forall y \geq 0$ , and  $y \neq 0$ ,  $0 \notin L(y)$  and  $L(0) = \mathfrak{R}_+^N$
- L2. if  $x \in L(y)$ ,  $\lambda x \in L(y)$ ,  $\forall \lambda \geq 1$
- L3.  $\forall x \in L(y)$ , if  $x' \geq x \Rightarrow x' \in L(y)$
- L4.  $L(y)$  is a closed set,  $\forall y \in \mathfrak{R}_+^M$
- L5.  $L(\theta y) \subset L(y) \quad \forall \theta \geq 1$ , or  $L(\theta y) = \theta L(y) \quad \forall \theta > 0$
- L6. if  $y' \geq y \Rightarrow L(y') \subset L(y)$

The L1 axiom refers to the no free lunch concept, i.e., it is impossible to produce some positive output from a zero value of input. The strong input disposability is modelled by two axioms L2 and L3. Axiom L4 implies the existence of isoquants at the boundary of the feasible set. L5 states that a proportional increase of output could be generated by a more important input requirement set. Property L6 clarifies the strong version of L5 and implies that the increased output is not necessarily proportional. The crucial assumption on the traditional convexity is not added to the list of axioms satisfied by the input set  $L(y)$ . Out of this defined set, a technology can be conveniently represented by the input directional distance function. The technical efficiency input oriented is measured using this function, defined as:

$$\bar{D}_i(x, y; g_x) = \max_{\theta} \left\{ \theta \mid (x - \theta g_x) \in L(y) \right\}$$

This function satisfies the proprieties given by Chambers et al., (1998) and Färe et al., (2004).

Next we'll show that a natural definition of FDH technology is given by the individual production requirement set, obtained from only one production unit  $(x_k, y_k)$ , under free disposability and variable returns to scale:  $T^k(x_k, y_k) = \{(x, y) \mid x \geq x_k, y \leq y_k\}$ . The non-convex technology is the union of this individual production set and it is defined as:

$$T_{FDH} = \bigcup_{k=1}^K T^k(x_k, y_k)$$

From the definition of the  $k$  subsets, we have the following definition of FDH technology:

$$T_{FDH}(x_k, y_k) = \left\{ (x, y) \mid \exists z_k \geq 0, \sum_{k=1}^K z_k = 1, z_k x \geq z_k x_k, z_k y \leq z_k y_k, \forall k = 1, \dots, K \right\} \quad (2)$$

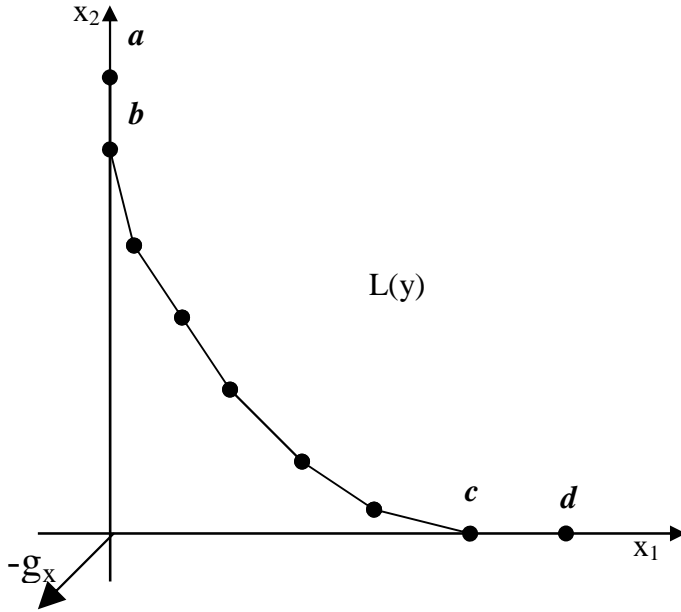
Where  $z$  is a vector of activity. This expression is the operational definition of the input requirement set, based on the FDH technology. Regarding the convex technology, the non-convex one is the smallest approximation of the true production frontier (Briec et al., 2004). This can also be phrased in terms of the so-called minimum extrapolation principle (Banker et al. 1984; Bogetoft, 1996). The minimum extrapolation principle guarantee that  $L(y)$  is closed (axiom L4).

According to the non-parametric approach, the efficiency measurement usually reveals two problems. The first one concerns the production frontier estimation, which is solved by using FDH technology (Cherchye et al., 2001). The second problem is related to measuring the distance between the valued production units and its reference unit on the frontier production. Here, we use the input directional distance function instead of the classical Shepard's distance function. By using the directional function, we can aggregate the inefficiency patient scores on the ward level and directly compare these wards' inefficiencies scores.

However, the input directional function has a major inconvenience, especially for the purpose of the measurement of input efficiency at the patient level. This inconvenience is related to zero input values. In the patient data set, there are many patients who do not use some of the inputs every day during his/her hospitalization. In this setting, observed patients with no input use will be efficient in respect to the input directional distance function. This case can be deduced from the function definition:  $\bar{D}_i(x, y; g_x) \geq 0$  and if the input direction is

nonnegative ( $g_x \succ 0$ ) then  $\bar{D}_i(x, y; g_x) = 0$  for zero input value. Figure 1 proves that input directional distance function generate efficient units when no inputs are used. The production units  $a$  and  $d$  have at least one zero input value and are found efficient by a directional distance function but they are inefficient if a radial measure is considered.

Figure 1 Problem of zeros with input directional distance function



To avoid this problem, we introduce a non-radial measure of efficiency into the directional input function (1). This function coupled with the non-radial measure provide separate contraction of each input ( $\theta_n$ ) instead of a common  $\theta$  contraction, as defined below in the expression (3). The  $\theta_n$  value will be specified for each input. Färe and Lovell (1978) ( $FL(x, y)$ ) proposed this non-radial measure of technical efficiency which scale each input individually by different proportions so that they can be projected to the efficient frontier. The input based on non-radial measure is defined as follow:

$$FL(y, x) = \min \frac{\sum_{n=1}^N \theta_n}{\sum_{n=1}^N \delta(x_{ni})} \left| (\theta_1 x_1, \dots, \theta_n x_n, \dots, \theta_N x_N) \in L(y), \theta_n \in (0, 1], \forall n = 1, \dots, N \right. \quad (3)$$

where  $\delta(x_n) = 1$  if  $x_n > 0$   
 $\delta(x_n) = 0$ , otherwise

The  $FL(x, y)$  measure minimizes the arithmetic mean of the (scalar  $\theta_n$ ) as a proportional reduction for each input, separately. For an observation  $(x_k, y_k)$ , the projection point  $(x_k^*, y_k^*)$  is determined by scaling down each input by the corresponding element of the efficient

measure  $(x_k^*, y_k^*) = (\theta_1 x_{k1}, \dots, \theta_N x_{kN}, y_{kN})$  and will always belong to the Pareto-Koopmans efficient subset of  $L(y_k)$ .

Each input is then contracted in a non-radial manner according to its own direction. In this respect, we will be able to aggregate efficiency score at the ward level. Hence, we adapt this measure by the input directional distance measure in order to get aggregation:

$$\begin{aligned} \vec{D}_i(x, y; g_x) = \\ \min \left\{ \frac{\sum_{n=1}^N \theta_n}{\sum_{n=1}^N \delta(x_n)} \mid (x_1 - \theta_1 g_1, \dots, x_n - \theta_n g_n, \dots, x_N - \theta_N g_N) \in L(y), \theta_n \in (0, 1], \forall n = 1, \dots, N \right\} \\ \text{where } \delta(x_n) = 1 \text{ if } x_n > 0, \delta(x_n) = 0, \text{ otherwise} \end{aligned} \quad (4)$$

In its original version, the directional Färe-Lovell function (expression 4) uses the arithmetic mean to compute the minimal contraction on each input; however, we prefer to minimize a weighted average. Note that a potential problem of the  $FL(x, y)$  measure is the implicit assumption that all inputs have equal input weights. Here, we use the share of input costs on the amount of total expenditures as the weighting element ( $v_n$ ). Finally, we get the  $FL(x, y)$  measure as follows:

$$\begin{aligned} \vec{D}_i(x, y; g_x) = \\ \min \left\{ \frac{\sum_{n=1}^N v_n \theta_n}{\sum_{n=1}^N \delta(x_n)} \mid (x_1 - \theta_1 g_1, \dots, x_n - \theta_n g_n, \dots, x_N - \theta_N g_N) \in L(y), \theta_n \in (0, 1], \forall n = 1, \dots, N \right\} \\ \text{where } \delta(x_n) = 1 \text{ if } x_n > 0, \delta(x_n) = 0, \text{ otherwise} \end{aligned} \quad (5)$$

The weighted element is summed to unity ( $\sum_{n=1}^N v_n = 1$ ).

The last step consists of the linear program which allows us to estimate this directional distance function for a given FDH technology. Following the two recent development, as shown by Agrell and Tind (2001) and Leleu (2006), the FDH technology can be written using a linear program including the directional efficiency measure. This linearization allows the duality and the economic interpretation in shadow prices terms of the FDH technology.

Using the definition of the FDH technology (2) and the non-radial input directional distance function (5), the linear FDH model to gauge technical efficiency is defined:

$$\begin{aligned}
\bar{D}_i^{FDH}(x_{k'}, y_{k'}; g_x) &= \max_{\theta_{kn}, z_k} \sum_{n=1}^N \sum_{k=1}^K v_n \theta_{kn} \\
\text{s.t. : } z_k (x_{kn} - x_{k'n}) &\leq -\theta_{kn} g_x, \quad n = 1, \dots, N \\
z_k (y_{km} - y_{k'm}) &\geq 0, \quad m = 1, \dots, M, \\
\sum_{k=1}^K z_k &= 1, \quad z_k \geq 0, \quad k = 1, \dots, K \\
\theta_{kn} &\geq 0, \quad n = 1, \dots, N, \quad k = 1, \dots, K
\end{aligned} \tag{6}$$

### 3. Data

We use the linear FDH model (6) to estimate scores of technical efficiency at patient level and to aggregate the obtained scores at ward level. Required data fully describing the production process would be necessary for an empirical specification of the production technology. In Tunisia, it is not easy to get this kind of medical and economic data. Therefore, in this study, we obtain data that was available from two sources for the year 2002. Patient level data were retrieved from the hospital morbidity and mortality survey (National Institute of Public Health, 2002) and completed by additional information obtained from the patient's bill. Data elements obtained from the survey include patient's socio-economic characteristics (age, sex, residence, etc.) and medical parameters (main and secondary diagnosis, discharge health status- dead or alive-). The patient billing system provides information on the quantity of medical acts, length of stays, as well as their corresponding expenditures. The data file provides the specification of the production technology, as designed by the medical staff.

As stated above, we define three inputs and one output in order to describe the input oriented production technology. The inputs are biological assessments (B), specialized medical care (KE), and inpatient days (ID). Information on these inputs is available in quantity and monetary units, as defined in the Tunisian official nomenclature book (Tarifs et nomenclature des actes professionnels, 1995). Each medical act is designated by a key letter and a coefficient: (i) KE include all specialized acts achieved by the physician following his expertise (ii) B design all laboratory acts achieved by authorized biologist or pharmacist.

On the cardiology ward activities, KE include all cardio-vascular acts.

They are expressed on monetary unit respectively to 0.16 TND<sup>1</sup> for B, 1.2 TND for KE, and 35.0 TND for ID. The ID is evaluated as an input proxy or day's expenditures, including medical and nursing staff, hostelry fees and medical disposals.

We define our single output as the patient discharge health status in its two modalities: alive and dead. The first modality means health status improvement, assuming that if the patient leaves the hospital alive, his/her health status has improved due to the treatment received at the ward level.

To ensure appropriate benchmarking, our efficiency analysis, included monitoring variables-major diagnoses, cardiovascular surgery acts and age - guaranteeing that the calculation of the efficiency scores at patient level belong to groups having similar diagnosis (the hospital morbidity and mortality survey encoding are based on the international classification of diseases-ICD10), as well as age of the patient.

File data show that the five wards, in our study can be counted in physical units as well as redefined according to the Tunisian nomenclature: 58 9178 B acts on B, 36087 KE acts and 7224 ID. They have offered these medical care's to 4878 patients who have 217 diagnoses as the ICD-10.

Table 1 Description statistics for the three inputs (unit: TND)

	Inputs	Sum	Mean	Min	Max
W1	B	157 394,160	239,930	0	2 416,800
	KE	54 457,600	83,0150	0	2 074,800
	ID	256 807,504	391,475	0	2 520,000
W2	B	56 759,400	55,650	0	752,000
	KE	52 938,200	51,900	0	595,000
	ID	376 895,834	369,506	0	9 590,000
W3	B	70 551,52	81,374	0	572,800
	KE	85 036,65	98,082	0	520,800
	ID	301 605,317	347,872	11,667	4 305,000
W4	B	34 157,00	21,402	0	349,600
	KE	230 389,60	144,354	0	6 067,20

<sup>1</sup> TND : Tunisian National Dinars

	ID	73 770,864	462,275	0	17 200,000
	B	94 268,48	127,735	0	1 842,40
W5	KE	433 044,4	58,678	0	5 487,6
	ID	252 381,251	341,98	0	3 325,00

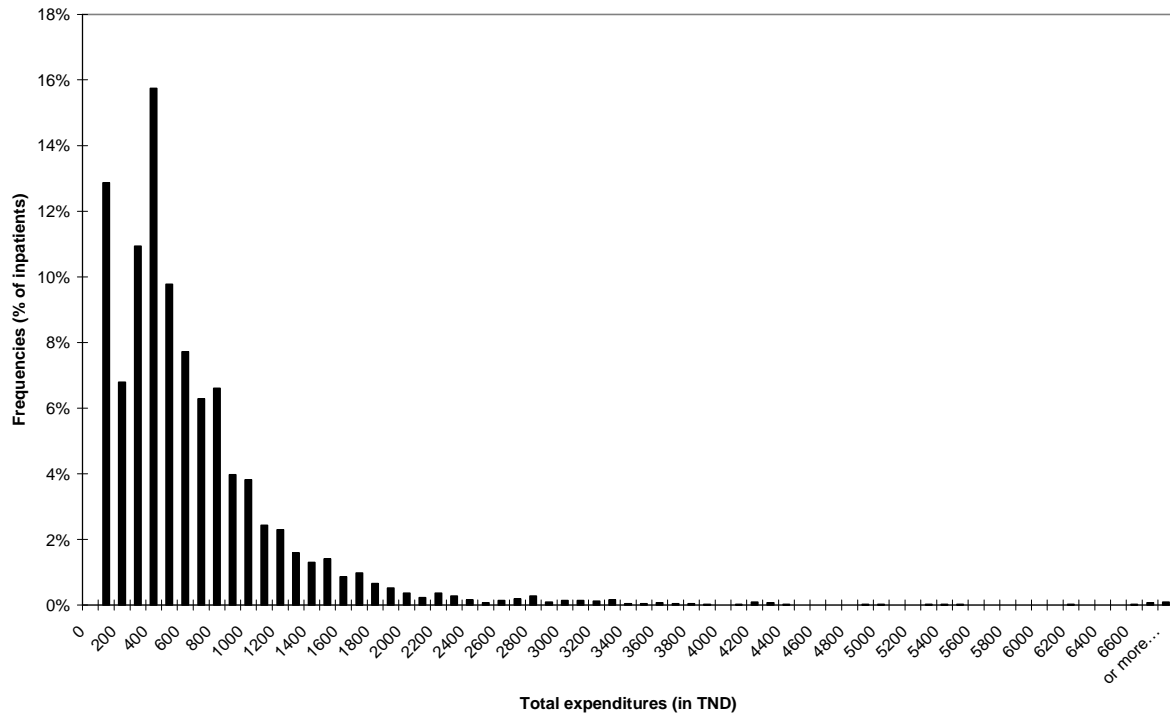
In some cases, we found some patients were admitted for one day of medical monitoring, but did not receive any formal therapeutic care. Therefore, these observations (inpatients) could overestimate the technical inefficiency score, and should be considered as outliers. Indeed, we found that such patients are always efficient.

The total expenditures diagram (figure 2) allows detection the problem of outliers. The first stick of the diagram reports sums one day's inpatient expenditures, which define the outliers.

Production units with an extreme production structure may be declared as efficient simply because of its special production structure. Possible outlier influence is increased on the non-parametric techniques, implying that even measurement error can have significant influence. The problem of non-similarity and outlier influence implies that it is not possible to achieve a complete ranking of the production units because many will be characterized as efficient. In general, there is a trade-off between a realistic description of the production profile and a complete ranking. If the efficiency analysis is based on a few numbers of variables then it is likely that a complete ranking can be obtained. However, restricting the number of variables to describe the production process might not provide a realistic impression of the production activities. On the other hand, inclusion of many variables will provide a more reliable description of the production activities, but this increases the possibility for specialisation and therefore makes a complete ranking less likely.

To avoid the outliers problem in this study, we eliminate the first quartile of the patient sample (patients set of the first stick), which may be equivalent to bootstrapped procedures applied to FDH estimators (Wilson and Simar, 2000).

Figure 2 Distribution of total expenditures among inpatients



#### 4. Results

An important issue of the inefficiency analysis is not only to determine the inefficiency levels but also to be able to explain the variation with reference to the characteristics of the production units. This section sets out the inefficiency scores obtained from using the linear FDH model (6). We organize it into three sub-sections. In the first sub-section we present brief results, according to the estimated and aggregated technical inefficiencies scores for each ward and for each input. We report on the second sub-section the concentration analysis of technical inefficiency for the three inputs. The concentration analysis could improve the identification of the inefficient disparities by wards at patient's level. In our third sub-section, we summarize the results of the wards' inefficiencies by major diagnosis. In fact, we assume that it would be significant to analyze wards inefficiencies, according to major diagnoses patients, as inefficient sources.

The non-radial Färe-Lovell efficiency measure contains detailed information concerning the performance of each of the included inputs. For example, the input-oriented Färe-Lovell measures will not only provide an overall efficiency score but also determine the input specific efficiency scores. The Färe-Lovell measure is then calculated as the average of these individual efficiency scores.

##### 4.1 Ward Inefficiencies

The estimated results are summarized in table 2 below. This table gives the average inefficient scores at wards level. Note the large disparities in the inefficiency of cardiology wards which if eliminated could result in the potential savings in the input set. We start by analyzing the inefficiencies' variability related to each ward.

For the three inputs, the average technical inefficiency for the five wards in our study was at least 54%, meaning that for the five wards inputs were inefficiently employed. The minimum value of inefficiency is 31% (W5; input B), while, the maximum value is 80.4% (W4; input B).

W1 accounts for 568 admitted patients; this ward has an inefficiency score of 55.3% for the input B, 57.5 % for the KE input and 52.1 % for the ID input. W2 admitted 882 patients and its inefficient scores are respectively 68.8%, 74.2 % and 55.8 % for the three inputs. This second ward is more inefficient than W1 in the three inputs. W3 differs in term of variability, given approximately the same inefficient values obtained for the three inputs; these values are equal to 56%. This ward has recorded 565 inpatients. W3 is less inefficient than W2; however it has the same values as the W1.

For the three inputs, the distribution of inefficient scores is different for W4. This ward was the most inefficient regarding the utilization of the input B (80.4 %) and the least inefficient for the input KE (40.6%). The FDH results for W5 show respectively an inefficient score of 30.1% for the input B, 42.7% for the input KE and 50.5% for the input ID. W5 could be considered as relatively the most efficient ward on all inputs used, except for KE input, where it should be ranked second after W4.

Table 2 Average technical inefficiencies for each ward by input types

Wards	Inefficiency B	Inefficiency KE	Inefficiency ID
W1	55.26%	57.51%	52.09%
W2	68.85%	74.20%	55.84%
W3	56.02%	55.85%	56.94%
W4	80.38%	40.65%	56.60%
W5	30.93%	42.66%	50.54%
Total	58.29%	54.17%	54.40%

Furthermore, while examining table 2, disparity appears among wards inefficiencies linked to inputs B, KE and ID. For the three inputs considered in this study, the potential saving is at least 31 %.

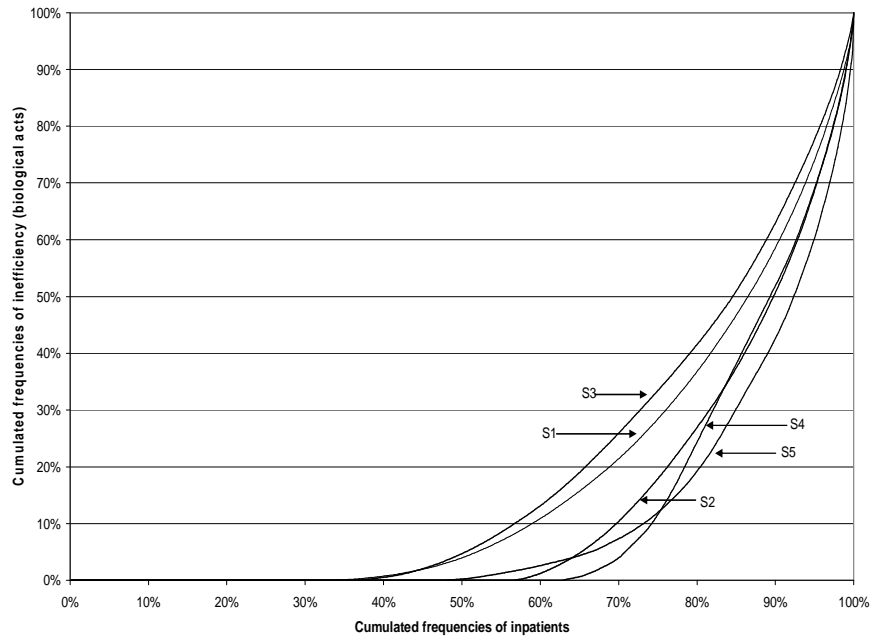
It is clear that health care delivery in cardiovascular wards do not efficiently use their inputs. This may be related to the production process. Table 2 shows that the inefficiencies are related to all three inputs, averaging slightly more than 50% inefficient. Inefficiency is quite heterogeneous as observed for the inputs B and KE, given the higher inefficient levels at W2 for both inputs and W4 for B inputs. Conversely, W5 was the most efficient in its use of the three inputs. W4 was the most inefficient in its use of input B (80.38%). W2 was the most inefficient in its use of input KE (74.20%). These inefficiency scores of wards W1 and W3 were roughly similar for the three inputs and their value is at least 52%. W4 is less inefficient in KE inputs (40.65 %). The ID inefficiency distribution is homogeneous. The inefficient scores are important (55 %). We find that W3 presents the highest inefficiency and W5 give the less inefficient (see table 2).

#### **4.2 Concentration analysis**

For the five wards, concentration analysis can explain more the source of input inefficiencies regarding treated patients. We perform this analysis for the three inputs and for all inpatients. To carry out this concentration analysis, we report in the graph the cumulative share of inpatients on the horizontal axis, and the cumulated frequencies of the input inefficiency on the vertical axis.

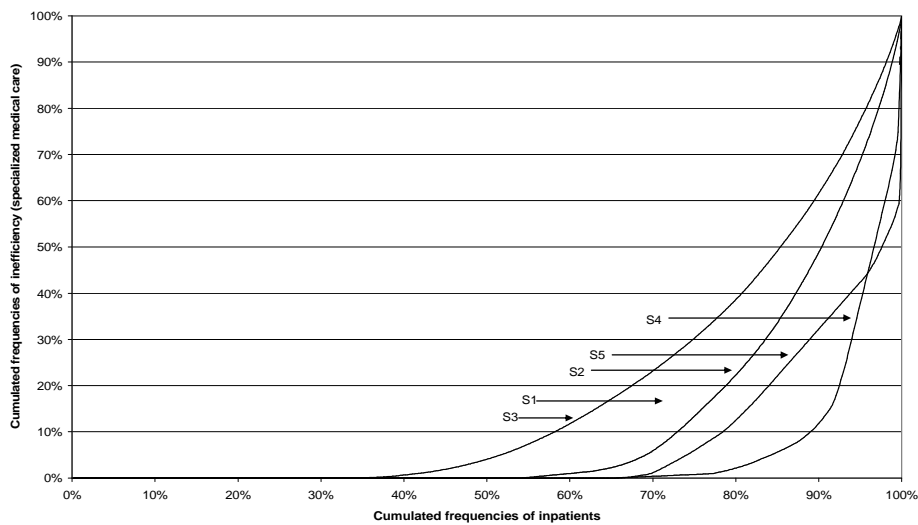
The first curve (figure 3) represents the concentration of the input B inefficiencies distribution among inpatients. It shows that there is a high concentration for wards W4 and W2 and an average concentration for the remaining wards. The technical inefficiencies recorded for W4 and W2 are concentrated respectively on 35% and 40% of total inpatients. For W1 and W3, we can see that 100% of technical inefficiencies are concentrated on roughly 40% of inpatients. 50 % of patients admitted at W5 generate the 100% of technical inefficiencies. For input B, the inefficiencies at the ward level should be related to some treated patients, we also find that these inefficiencies are due to approximately 50% of inpatients.

Figure 3 Concentration of inefficiency on biological acts (B)



The second concentration curve (figure 4) is linked to KE input. Here, we find that for W4, only 30% of inpatients caused 100% of inefficiencies and 90% of inefficiencies are concentrated among 10% inpatients. The situation is not the same for W1 and W3, where 60% of their inpatients accounts for their total inefficiencies. W5 provides another view of the concentration analysis. Its inefficiency is caused by only 30% of inpatients and 80% of this ward inefficiency is related to only 10% of inpatients. The W5 shows that the 100% of the technical inefficiencies is concentrated among 50% of inpatients.

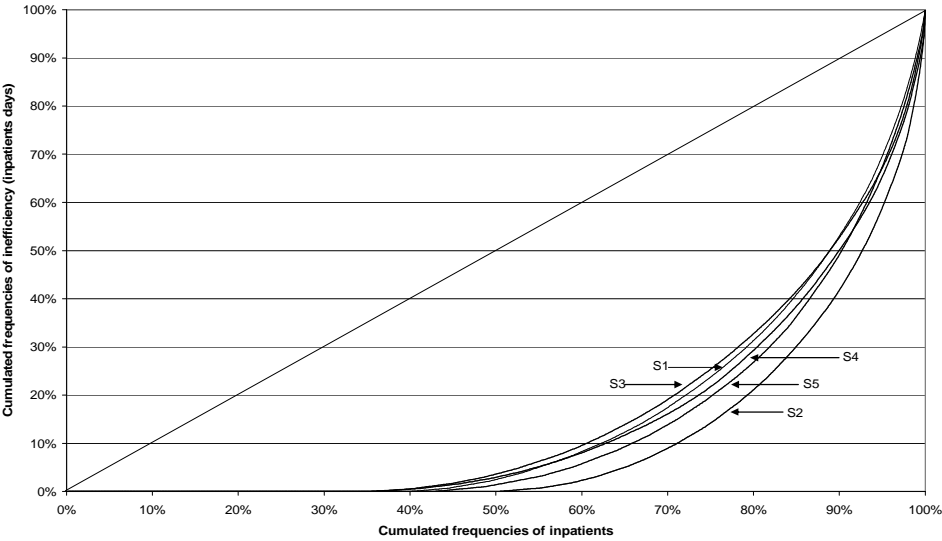
Figure 4 Concentration of inefficiency on specialized medical care (KE)



The last concentration curve (figure 5) allows us to analyze the inefficiencies concentration of ID input among wards inpatients. The concentration analysis shows the 50 % of inpatients

explain the 100% of the W2 inefficiencies. Therefore, only 60 % of inpatients explain to total inefficiencies for the remaining wards.

Figure 5 Concentration of inefficiency on inpatients days (ID)



**4.3 Wards inefficiencies by major diagnoses**

As for the high level of inefficient scores for all wards and the strong concentration among inpatients, we assume that inefficiency should be related to fewer but the same diagnoses for inpatients. The patients’ diagnostics are coded according to the 10<sup>th</sup> international classification of diseases (ICD-10). For the five cardio-vascular wards, we find 217 major diagnoses, but that only 50 diagnoses really implicated as sources of inefficiency for all the wards. The distribution of the input based inefficiency scores by wards and by major diagnoses is useful to identify sources of inefficiency. There’s a difference on inefficiencies scores according to diagnoses. The selected diagnoses are implicated in the estimated scores of inefficiencies; explaining 95% to 98% of wards inefficiencies. We observe some diagnoses sources of inefficiencies for same words but not for others ones. Table 3 below; summarize the disparities on the implication of diagnoses on wards’ inefficiencies. 39 diagnoses are implicated on more than 95% of W1 inefficiencies for the three inputs. For the W2, there are at least 22 diagnoses sources of 97% of its inefficiencies. 32 diagnoses are at least sources of 95% of W3 inefficiencies. W4 accounts for only 14 diagnoses as sources of 98 % of its inefficiencies.

Table 3 Number of diagnoses as sources of inefficiencies for each ward by the three inputs

Wards	W1	W2	W3	W4	W5
Input B	38 (95.4%)	25 (98.3%)	36 (97.3%)	14 (98.3%)	28 (95.5%)

Input KE	39 (98.2%)	24 (97.9%)	34 (95.3%)	15 (99.7%)	26 (96.9%)
Input ID	38 (96.8%)	22 (97.5%)	32 (94.9%)	15 (98.6%)	29 (95.8%)

The distribution of inefficiencies in the input B shows that the diagnoses Unstable Angina (UA) and Acute Myocardial Infraction Unspecified (AMIU) are the main sources of inefficiency for wards W1, W2 and W4. We find that a patient treated for UA diagnostic contribute specifically to the high inefficiencies at W1, W2 and W3. If we add the second diagnoses, the related inefficiencies are 32.4% for W1, 34.3 for W2 and 62.9% for W4. We also observe that 95.4 % of the inefficiency at ward W1 is explained by 38 diagnoses, where 50.3 % is due to patients treated with the following four diagnoses: AU, AMIU, Essential Hypertension (EP) and Atrial Fibrillation and Flutter (AFF). Therefore, the empty cell can be informative regarding the inefficiencies concentration by major diagnoses. For instance, 98.3% of the W4 inefficiency is related to only 14 diagnoses, and 62.9 % of its inefficiency is only due to only two diagnoses. The W3 records 58.2 % of its inefficiency on treating patients for the five following pathologies: AMIU, EP, AFF, Chest pain, unspecified, and unavailability and inaccessibility of other helping agencies.

The wards' inefficiency related to the KE input confirms the implication of two types of diagnoses (UA and AMIU) in the higher inefficiency scores for the wards W1, W2, W3 and W4 but not for W5. Together UA and AMIU diagnoses contribute by 33.4% of the W1 inefficiencies, 36.5 % of the W2 inefficiencies, 18.6% of the W3 inefficiencies, 55.2% of the W4 inefficiencies, but to only 3.0% of the W5 inefficiencies. For W5, other non-rheumatic mitral valve disorders diagnosis is the main source of its inefficiency (35.0%). Nevertheless, this diagnosis does not add to the inefficiencies experienced in W2 and W4 and low values for W1 (0.3%) and W3 (0.5%).

For the third input (ID), the diagnoses AU and the AMIU remain the main sources of the technical inefficiency for W1, W2 and W4 but not for W5. These diagnoses represent respectively 26.7% of the W1 inefficiency, 31.9% of the W2 inefficiency, 10.6% of the W3 inefficiency and 63.6% of the W4 inefficiency. However, these diagnoses accounted for 2.0% of W5's inefficiency. The essential hypertension diagnosis was the main source of inefficiencies for W5 (26.6%) and contributes also to the technical inefficiency for W3 (14.5%) and W2 (11.5%). Thus, other forms of acute ischemic heart disease diagnosis represents 10.7 % of W5 inefficiency but at lower values for the other wards. The AFF diagnoses are consistently present as an inefficiency source in all five wards.

## 5. Discussion

The results reported in this paper make an important contribution to learn about sources of inefficiency on cardiology's wards, dealing with the benchmarking analysis of FDH model.

Results obtained regarding ward inefficiencies by input types show that the W2 and W4 have been shown as the most inefficient users of the B input. W2 is the most inefficient in the KE input. These two wards are located in the largest university hospitals that are able to provide all medical acts needed, meaning that patient transfers to other health facilities are usually unnecessary. For instance, the staff number in ward W2 is twice as those working in the other wards.

W4 is less inefficient in KE inputs (40.65 %), which may be a function of the proximity to a highly specialized ward (within the same building), exclusively devoted to cardiology medical acts (functional exploration and hemodynamic).

W5 seems to be more technically efficient than other wards, which may be due to its recent creation and reduced medical staff and equipment. Moreover, physicians from other health facilities usually do not refer complicated cardiology diseases to this ward. While W2 and W4 are particularly concerned with patients who are admitted as surgical cases, we believe that medical acts could be repeatedly ordered for the same patients, by different medical staff members (residents, assistants, professors, etc.) indicating a lack of coordination. Furthermore, we note that medical staff on cardiology wards uses the input KE differently, given the specialized medical practices multiplies process for the same diagnoses diseases.

For the inpatient days input, all wards have recorded higher inefficiency scores, which vary from 50% to 57%, leading to the potential saving by reducing the length of stay by half.

The disparity of wards technical inefficiencies scores suggest key role for factors like wards missions, inpatients health status, patients' profiles and illness severity. The concentration analysis shows more concentration prevailing in W2 and W4 than in W3 and W1. This disparity may be due to a higher amount of heterogeneity in the kind of biological assessments realized by the W2 and W4 laboratories, which are better at assessing a wide variety of biological exams. The W3 inpatients show a more concentration distribution of inefficiency than W4 or W2. Also, the observed concentration in W2 and W4 could be

explained by each ward's ability to assume more sophisticated specialized medical acts than the other wards.

We have also shown that the two diagnoses (UA and AMIU) are mainly involved in the elevated scores of inefficiency observed in biological acts, specialized acts as well as in inpatients days. These higher scores seem to be linked to the three prestigious cardiology wards: W1, W2 and W4. The patients treated with those diagnoses contribute by a lower value of inefficiency for ward W3 and an even smaller value for W5 (see table 4 below) which are smaller

Table 4 the main diagnoses sources of wards inefficiency

Diagnoses	Inputs	W1 Inefficiency	W2 Inefficiency	W3 Inefficiency	W4 Inefficiency	W5 Inefficiency
UA	B	21.9	26.2	65	29.1	0.3
	KE	22.8	28.9	7.9	30.4	0.4
	ID	18.2	24.0	3.2	34.8	-
	Total	62.9	79.1	17.6	94.3	0.7
AMIP	B	10.5	8.1	11.4	33.8	5.4
	KE	10.6	7.6	10.7	24.8	2.6
	ID	8.5	7.9	7.4	28.8	2.0
	Total	29.6	23.6	29.5	87.4	10.0

In comparison with wards W1, W2, W4, the recently created W5 shows a very lower score of inefficiency for such complicated diagnoses. On the other hand, it becomes clear that the W5 has some difficulty addressing common non-treated diagnoses such as elevated blood pressure (HTA) (rated 26.6% vs. only 0.5% at W4 in terms of inpatient days inefficiency) or some other non-rheumatic mitral valve disorders (rated 35% vs. 0.3% at W1) in terms of specialized medical care acts inefficiency.

So why this disparity? We could suggest that the W5 does not have the abilities and the qualified staff to take care of the acute and complicated cases, such as AMIU and AU, which represent a highly life threatening diagnoses, and are supposed to be treated in those confirmed and notorious wards, as W2 or W4. Therefore, a high score of inefficiency observed at W4 cardiology may be due to a high rate of inpatients recruitment and severe cases, than to the lack of performance.

This result confirms the observed relationship between the high level of technical inefficiency and some diagnoses. At the patient level, inefficiency could be reduced, by improving the management care of patients with those pathologies. This kind of observation and analysis may be of major interest among comparable wards, on basic criteria such as wards missions, staff qualification and effective operational capabilities. So, it may also depend on the health priority design of the medical staff, in their choice of inpatients recruitment.

The results reported in this study demonstrate the existence of technical inefficiency in Tunisian Public Hospitals. The various groups of inpatients may explain the results for the variability in hospitals' performances. Disparity of wards technical inefficiencies scores suggest key role of factors, like wards missions, the illness severity and the inpatients profiles. This study's results show that the government must not only pursue cost containment policies but it should also focus on more efficient production of hospital wards, in order to obtain more benefits for the same investment.

Cardiovascular wards differ in their performance, and it is not easy to explain this disparity, without a diagnosis related groups and medical production processes analysis. We believe that clinical practice is quite different, even when physicians are treating the same case-mix patients, each performing small numbers of widely different procedures. We suggest that the wards' inefficiencies were related to the organization of care. Our study shows inefficiencies regarding inpatients days, gives rise to a pattern of production processes. Regarding the wards functioning, two specific issues arise. First, one issue pertains to the optimum size for a particular health care unit and how services should be organized to ensure a greater level of efficiency. Second, whether there are aspects of organization that have an impact on health care delivery which needed to be elucidated

Given the Tunisian public hospitals context, optimal size leads to greater ward efficiencies. For a wide range of wards interventions, there is a clear relationship between volume and outcome i.e., wards treating more patients provide better treatment; whether through practise or the availability of standardized routines, better equipment, or some other factor. Wards could also obtain better results because they treat less seriously diseases, without UA and AMIU pathologies. The first arguments explain the higher inefficiencies for wards W2 and W4 and the latter one justifies the less inefficiency for W5.

This investigation suggests that those factors, leading to a ward being deemed inefficient in organizational terms may also contribute to bad quality of care. Importantly, this study indicates that, when factors such as ward missions, illness severity, junior medical staff, and ward size were taken into account, there were significant inputs saving (medicals acts, length of stay) and reducing variation in outcome.

## **6. Conclusion**

In this paper we present the results of an analysis of efficiency patterns for Tunisian's cardiology wards using the non-parametric techniques of the FDH. The paper has demonstrated that it is feasible to use this technique to examine the productive performance of cardiology wards. In particular, the application has shown that linear FDH technology can provide useful information regarding the efficiency patterns. This information relates both to the wards as well as to the patients' level. In the Tunisian's public hospital system, a relatively high inefficiency level was detected. Obviously, the efficiency results depend on the technology assumption used.

The principal contribution of this paper is the development of the linear FDH model to measure non-radial directional input inefficiency. The technical inefficiency measurement and analysis concern the cardiovascular wards of Tunisian public hospitals. The inefficiency measures are achieved with a directional non-radial oriented input. The major novelty of this paper is that it uses jointly a non-radial directional efficiency measured at patient level, using the recently implemented linear non-convex model (Agrell and Tind, 2001; Leleu, 2006). Using such a model allows us to aggregate inefficiencies at the ward level. We also argue that this is the first time, when an evaluation of technical inefficiency has been performed for Tunisian public hospitals.

In the empirical part of this paper, we evaluate inefficiency using one output and three inputs. The FDH Results are presented according to aggregated wards inefficiencies, concentration analysis and distribution of the wards inefficiencies by inputs and by major diagnoses.

Tunisian public's hospitals are changing, and facing the demographic and epidemiological transition, which constitute both opportunities and challenges. Cardiovascular diseases induce a high hospital cost. Reducing wards inefficiencies should be included as a priority issue to save resources. In this respect, there is a growing body of evidence that wards efficiencies could depend on the organization structures and size, but many questions remain without answers. In terms of used resources, the lower levels of efficiency seem to hinder achieving

the desired results. We need a better understanding of the relationship between ward inefficiency and the underlined factors, like wards missions, wards sizes (medical staff, equipment), inpatient illness severity, inpatient recruitment and wards outcome, meaning the relationship between wards inefficiencies and resource saving and improvement outcome.

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